Atomic levels in superstrong magnetic fields and $D = 2$ QED of massive electrons: screening

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plan

- $a_B, a_H,\ a_H << a_B \Longrightarrow B >> e^3 m_e^2$ electron from Landau level feels weak Coulomb potential moving along axis z; Loudon, Elliott 1960: $E_0 = -(me^4/2) \times \ln^2(B/(m^2 e^3))$?
- $D = 2$ QED - Schwinger model with massive electrons, radiative "corrections" to Coulomb potential in $d = 1$; $\Pi_{\mu\nu}$, interpolating formula, analytical formula for $\Phi(z)$, $g > m$ - photon "mass" $m_\gamma \sim g$, screening at ALL z when $g > m$
- $D = 4$ QED; photon "mass" $m_\gamma^2 = e^3 B$ at superstrong magnetic fields $B >> m_e^2/e^3 = 137 \times 4.4 \times 10^{13}$ gauss; asymptotic behaviour of $\Phi(z)$ at $z >> 1/m_e$ (no screening) and at $z << 1/m_e$ (photon "mass" and screening)
- ground state hydrogen atom energy in the superstrong magnetic field; excited levels

hydrogen atom in strong B

$$d = 3 : (p^2/(2m) - e^2/r)\chi(r) = E\chi(r)$$

$$R(r) = \chi(r)/r, r \geq 0, \chi(0) = 0$$

$$d = 1 : (p^2/(2m) - e^2/|z|)\Psi(z) = E\Psi(z)$$

$$-\infty < z < \infty, \ \ \Psi(0) \neq 0$$

variational method for ground state energy:

$$\Psi(z) \sim exp(-|z|/b);$$

$$< V >\sim \ln(1/\epsilon)$$

$d = 1 \Longrightarrow d = 3$ at $z < a_H \equiv 1/\sqrt{eB}$

$$V(z) = 1/\sqrt{z^2 + a_H^2}$$

$$\ln(1/\epsilon) \Longrightarrow 2\ln(a_B/a_H) = \ln(B/(m^2e^3))$$

$$E_0 = -(me^4/2) \times \ln^2(B/(m^2e^3))$$

first excited level: $\Psi_1(0) = 0, E_1 \Longrightarrow -me^4/2 \ (B \Longrightarrow \infty)$; degeneracy of odd and even levels; the only nondegenerate level - $E_0 \Longrightarrow -\infty$
Loudon (1959); A.N.Sisakyan...

Definitions (for this talk): $B > m_e^2e^3$ - strong B, $B > m_e^2/e^3$ - superstrong B.

superstrong B

QED loop corrections to photon propagator drastically change this picture for $B >> m_e^2/e^3$.
Dirac equation spectrum in a constant homogenious magnetic field looks like:

$$\varepsilon_n^2 = m^2 + p_z^2 + (2n+1)eB + \sigma eB \quad , \tag{1}$$

where $n = 0, 1, 2, ...$, $\sigma = \pm 1$ (Rabi, 1928,
$2n + 1 + \sigma \Longrightarrow 2j, \;\; j = 0, 1, 2, ...$)
$\varepsilon_n \gtrsim m/e$ - ultrarelativistic electrons; the only exception is the lowest Landau level (LLL) which has $n = 0$, $\sigma = -1$.
We will study states on which LLL splits in the field of nucleus.
Hydrogen atom: electron on LLL moves along axis z; proton stay at $z = 0$. What electric potential does electron feel? Let us look at $D = 2, d = D - 1 = 1$ QED.

$D = 2$ QED: screening of Φ

$$\Phi(\bar{k}) \equiv A_0(\bar{k}) = \frac{4\pi g}{\bar{k}^2} \; ; \quad \mathbf{\Phi} \equiv \mathbf{A}_0 = D_{00} + D_{00}\Pi_{00}D_{00} + \ldots$$

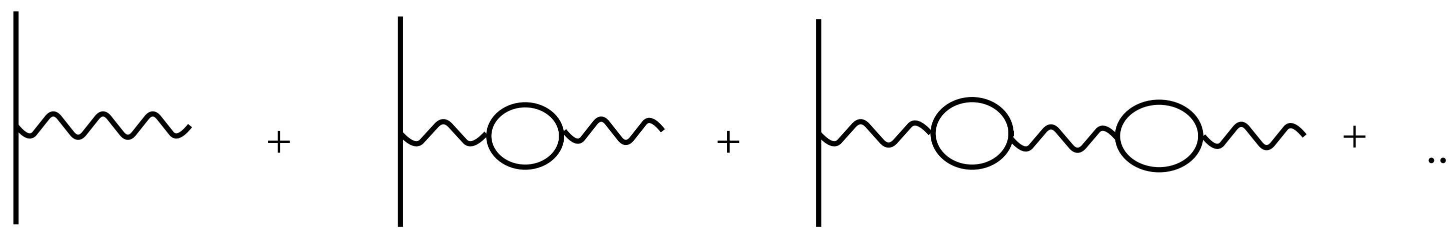


Fig 1. *Modification of the Coulomb potential due to the dressing of the photon propagator.*

Summing the series we get:

$$\mathbf{\Phi}(k) = -\frac{4\pi g}{k^2 + \Pi(k^2)} \; , \quad \Pi_{\mu\nu} \equiv \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \Pi(k^2) \qquad (2)$$

$$\Pi(k^2) = 4g^2 \left[\frac{1}{\sqrt{t(1+t)}} \ln(\sqrt{1+t} + \sqrt{t}) - 1 \right] \equiv -4g^2 P(t) \quad , \tag{3}$$

$$t \equiv -k^2/4m^2$$

Taking $k = (0, k_\|)$, $k^2 = -k_\|^2$ for the Coulomb potential in the coordinate representation we get:

$$\mathbf{\Phi}(z) = 4\pi g \int\limits_{-\infty}^{\infty} \frac{e^{ik_\| z} dk_\|/2\pi}{k_\|^2 + 4g^2 P(k_\|^2/4m^2)} \quad , \tag{4}$$

and the potential energy for the charges $+g$ and $-g$ is finally: $V(z) = -g\mathbf{\Phi}(z)$.

Asymptotics of $P(t)$ are:

$$P(t) = \begin{cases} \frac{2}{3}t & , \quad t \ll 1 \\ 1 & , \quad t \gg 1 \quad . \end{cases} \tag{5}$$

Let us take as an interpolating formula for $P(t)$ the following expression:

$$\overline{P}(t) = \frac{2t}{3 + 2t} \quad . \tag{6}$$

We checked that the accuracy of this approximation is not worse than 10% for the whole interval of t variation, $0 < t < \infty$.

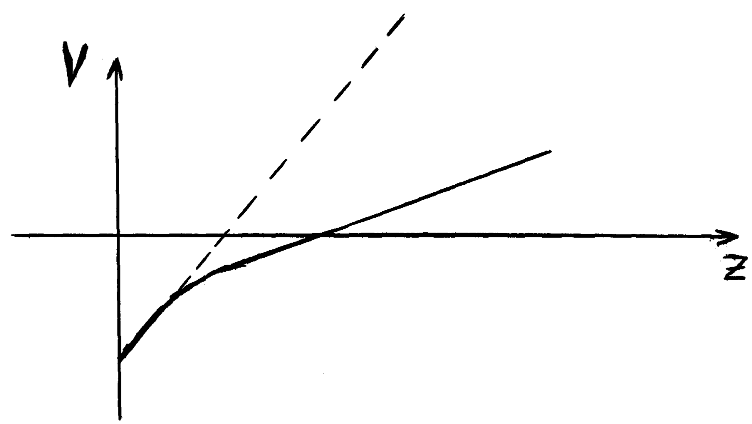
$$\begin{aligned}
\mathbf{\Phi} &= 4\pi g \int\limits_{-\infty}^{\infty} \frac{e^{ik_{\|}z} dk_{\|}/2\pi}{k_{\|}^2 + 4g^2(k_{\|}^2/2m^2)/(3 + k_{\|}^2/2m^2)} = \\
&= \frac{4\pi g}{1 + 2g^2/3m^2} \int\limits_{-\infty}^{\infty} \left[\frac{1}{k_{\|}^2} + \frac{2g^2/3m^2}{k_{\|}^2 + 6m^2 + 4g^2} \right] e^{ik_{\|}z} \frac{dk_{\|}}{2\pi} = \\
&= \frac{4\pi g}{1 + 2g^2/3m^2} \left[-\frac{1}{2}|z| + \frac{g^2/3m^2}{\sqrt{6m^2 + 4g^2}} \exp(-\sqrt{6m^2 + 4g^2}|z|) \right]
\end{aligned}$$

In the case of heavy fermions ($m \gg g$) the potential is given by the tree level expression; the corrections are suppressed as g^2/m^2.

In case of light fermions ($m \ll g$):

$$\Phi(z)\Big|_{m \ll g} = \begin{cases} \pi e^{-2g|z|} & , \quad z \ll \frac{1}{g}\ln\left(\frac{g}{m}\right) \\ -2\pi g\left(\frac{3m^2}{2g^2}\right)|z| & , \quad z \gg \frac{1}{g}\ln\left(\frac{g}{m}\right) \end{cases} . \qquad (8)$$

I am grateful to A.V. Smilga who noted privately that in the case of light fermions in $D = 2$ QED a massive pole in a photon propagator emerges.



$D = 4$ QED

$$\Phi = -\frac{4\pi e}{k^2 + \chi_2(k^2)} = \frac{4\pi e}{\left(k_\parallel^2 + k_\perp^2\right)\left(1 + \frac{\alpha}{3\pi}\ln\left(\frac{2eB}{m^2}\right)\right) + \frac{2e^3 B}{\pi}\exp\left(-\frac{k_\perp^2}{2eB}\right) P\left(\frac{k_\parallel^2}{4m^2}\right)} \quad (9)$$

Batalin, Shabad (1971), Shabad (1972,...); Skobelev(1975), Loskutov, Skobelev(1976): $B >> m^2/e,\ k_\parallel^2 << eB$

Linear in B term in photon polarization operator originates from the LLL parts of electron propagators. In coordinate representation transverse part of LLL wave function is: $\Psi \sim exp((-x^2 - y^2)eB)$ which in momentum representation gives $\Psi \sim exp((-k_x^2 - k_y^2)/eB)$.

Holding only LLL in spectral representation of electron Green functions and integrating their product over $dk_x dk_y$ we get factor eB, while longitudinal and time-like parts of propagators are that of free electrons.

Loskutov, Skobelev(1983); Kuznetsov, Mikheev, Osipov (2002): in superstrong B photon "mass" emerge.

$$\mathbf{\Phi}(z)\Bigg|_{|z|\gg\frac{1}{m}} = \frac{e}{|z|}\,, \quad V(z)\Bigg|_{z\gg\frac{1}{m}} = -\frac{e^2}{|z|} \qquad (10)$$

$$
\begin{aligned}
\mathbf{\Phi}(z)\Big|_{\frac{1}{m} \gg z \gg \frac{1}{\sqrt{eB}}} &= e\int_0^\infty \frac{\exp\left(-\sqrt{k_\perp^2 + \frac{2e^3B}{\pi}}|z|\right)}{\sqrt{k_\perp^2 + \frac{2e^3B}{\pi}}} k_\perp dk_\perp = \\
&= \frac{e}{|z|}\exp\left(-\sqrt{\frac{2e^3B}{\pi}}|z|\right) \quad , \\
V(z) &= -\frac{e^2}{|z|}\exp\left(-\sqrt{\frac{2e^3B}{\pi}}|z|\right) \quad . \qquad (11)
\end{aligned}
$$

atomic levels

$$E_0 = -2m \left(\int\limits_{a_H}^{a_B} U(z)dz \right)^2 \quad (12)$$

We split the integral into two parts: from $1/m$ to a_B, where the screening is absent (large z),

$$I_1 = -\int\limits_{1/m}^{a_B} \frac{e^2}{z} dz = -e^2 \ln\left(1/e^2\right) \quad (13)$$

and from the Larmour radius $a_H = 1/\sqrt{eB}$ to $1/m$, where the screening occurs (small z):

$$I_2 = -\int_{1/\sqrt{eB}}^{1/m} \frac{e^2}{z} \exp(-\sqrt{e^3 B} z) dz = -e^2 \ln(1/e) \quad . \qquad (14)$$

Finally we get:

$$E_0 = -(me^4/2) \times \ln^2(1/e^6) = -(me^4/2) \times 220 \qquad (15)$$

Freezing of ground state energy.
Without screening $I = -e^2 \ln(a_B/a_H)$,
$E_0 = -(me^4/2) \times \ln^2(B/m^2 e^3)$

Shabad, Usov (2007,2008). Analogous consideration to what I told for $D = 4$ + numerical estimates;
$220 \Longrightarrow 295; \;\; 15^2 \Longrightarrow 17^2$

Sadooghi, Sodeiri Jalili (2007) - $D = 4$, shape of potential, azimuthal asymmetry; dynamical mass of electron.

When B increases further Larmour radius approaches the size of a proton. This happens at $1/\sqrt{eB} \approx 1/m_\rho$, $m_\rho = 770$ MeV, $B \approx 10^{20}$ gauss. Taking into account the proton formfactor we get that for larger fields I_2 does not contribute to the energy, factor 220 should be substituted by 100: the ground level goes up.

Excited levels: corrections from screening should be larger for even states (Karnakov, Popov); degeneracy of even and odd states at $B \longrightarrow \infty$ occurs and is not influenced by screening (Loudon).

Conclusions

- ground state atomic energy at superstrong B - the only known (for me) case when radiative “correction” determines the energy of state
- analytical expression for charged particle electric potential in $d = 1$ is given; for $m < g$ screening take place at all distances
- asymptotics of potential at superstrong B at $d = 3$ are found confirming existing in literature results
- limit of ground state energy for $B >> m^2/e^3$ is determined analytically: $E_0 = -(me^4/2) \times \ln^2(1/e^6)$; $B > 10^{20}$ gauss: $e^6 \longrightarrow e^4$
- one more argument against existence of B_{cr}, at which upper and lower continuums merge