The transition $K^0 \to \bar{K}^0$ in the standard $SU(3) \otimes SU(2) \otimes U(1)$ scheme

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The $K^0 \to \bar{K}^0$ transition amplitude is calculated in the Kobayashi–Maskawa six-quark model with allowance for the strong interactions at small distances. The matrix element $\langle K^0 | H^{\text{eff}} | \bar{K}^0 \rangle$ is calculated by means of the vacuum insertion. Formulas are given for the mass difference Δm_{LS} of the K_L and K_S mesons and for the parameter ε of *CP* violation. The parameters of the six-quark model—the quark mixing angle θ_2 and the *CP*-odd phase δ—are determined for three values of the mass of the t quark ($m_t = 15$, 30, and 60 GeV).

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1. INTRODUCTION

The most popular model of the weak interaction at the present time is the six-quark generalization of the Weinberg–Salam scheme,[1] in which the charged currents are described by the Kobayashi–Maskawa matrix.[2] One of the virtues of this model is the natural appearance of the CP-odd phase in the matrix of the charged currents. In this paper, we consider the transitions $K^0 \to \bar{K}^0$ in the six-quark scheme with allowance for the strong interaction. We take into account the strong interaction in two stages: first, the gluon exchanges at small distances are summed in the framework of the renormalization group in the calculation of the coefficient function of the effective Lagrangian with $\Delta s = 2$; second, in the calculation of the matrix element of the effective Lagrangian between the K^0 and $\bar{K}^0$ mesons, we use the results of the vacuum insertion, which phenomenologically take into account the strong interactions at large distances. The $K^0 \to \bar{K}^0$ transition was considered in the four-quark model with allowance for the strong interaction at small distances in Ref. 3. In Ref. 4 the $K^0 \to \bar{K}^0$ transition was considered in the six-quark model with no gluon exchanges at small distances.

After calculating $\langle K^0 | -\mathscr{L}^{\text{eff}}_{\Delta S=2} | \bar{K}^0 \rangle$, we obtain expressions for the $K_L - K_S$ mass difference Δm_{LS} and for the parameter ε of CP violation in terms of the parameters of the six-quark model.

The violation of CP invariance in the Kobayashi–Maskawa model was considered in Refs. 5 and 6, and in Ref. 7 bounds were obtained on the difference between the parameters of CP violation in the decays $K_L \to \pi^+\pi^-$ and $K_L \to \pi^0\pi^0$ in this model. In Sec. 6 we show how the results of our work enable us to improve the accuracy of the numerical estimates of Ref. 7, and we present more precise values of $|\varepsilon'/\varepsilon|$.

The plan of the subsequent exposition is as follows. In Sec. 2 we recall the general properties of the Kobayashi–Maskawa model. In Sec. 3 the transition $d\bar{s} \to s\bar{d}$ is considered in the free-quark approximation. In Sec. 4 we consider the gluon corrections to the $d\bar{s} \to s\bar{d}$ transition and obtain the effective Lagrangian $\mathscr{L}^{\text{eff}}$ with $\Delta S = 2$. In Sec. 5 we calculate the matrix element $\langle K^0 | -\mathscr{L}_{\Delta S=2} | \bar{K}^0 \rangle$ and give expressions for Δm_{LS} and ε. Numerical estimates are presented in Sec. 6.

Let us indicate how our work differs from the papers of Ref. 4. In Ref. 4 the Lagrangian of the $d\bar{s} \to s\bar{d}$ transition was calculated in the free-quark approximation, whereas we take into account the effects of gluon exchanges at small distances in the leading logarithmic approximation. In the calculation of the matrix element $\langle K^0 | \mathscr{L}_{\Delta S=2} | \bar{K}^0 \rangle$, Shrock *et al.* used the result of the bag model[8]; Barger *et al.* considered two methods of calculating the matrix element, the bag model and the vacuum insertion, and they gave numerical estimates for both methods without assigning a perference to either of them. We argue that the most accurate estimate of the matrix element is given by the vacuum insertion. A numerical comparison of our results with those of Ref. 4 is given in Sec. 6.

2. THE KOBAYASHI–MASKAWA MODEL

Since the discovery of the J/ψ meson, four quarks (u, d, s, c) and four leptons (e, ν_e, μ, ν_μ) have been included in the weak interactions. The theory was then free of anomalies, and at high energies (much greater than the quark masses) the nondiagonal neutral current, which is absent in the bare Lagrangian but arises as a result of radiative corrections, was suppressed—the GIM mechanism[9] operated. Since then, there have been been discovered a third lepton, the τ, whose decays are evidently accompanied by the production of a third neutrino, the ν_τ, and also a new vector meson, the $\Upsilon(9.46)$, which consists of the quarks $b\bar{b}$ with charges $\pm\frac{1}{3}$. These new particles are most readily included in the previous weak-interaction scheme if it is assumed that there exists a sixth quark (the t) with charge $+\frac{2}{3}$, which appears in a single weak isodoublet with the b quark. In this case, the theory remains free of anomalies and is renormalizable. At the present time, it is known from Lederman's experiment on the t quark that, if this quark exists, its mass is greater than 10 GeV. This limit has been raised to 14 GeV in experiments at PETRA by Ting's group.

Let us consider the form of the charged currents in the six-quark model. The most general form of the currents is $q_A^+ U q_C$, where q_A^+ is the row of ano-quarks (u, c, t) and q_C is the column of catho-quarks (d, s, b);

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the Lorentz structure of the current is $\gamma_\alpha(1+\gamma_5)$. The matrix U is unitary. The unitarity of U is a consequence of two physical requirements: the universality of the weak interaction, $\sum_{k=1}^{3}|U_{jk}|^2=1$, and the suppression of the nondiagonal neutral currents due to the W-boson loops, $\sum_{k=1}^{3}U_{ik}U_{jk}|_{i\neq j}=0$. In the Weinberg-Salam model, the quark masses are equal to zero, and $U=1$, before the displacement of the Higgs field; then, with the development of the vacuum average, the quark fields become "distorted"—the states with definite mass are nondiagonal with respect to the weak interaction, and the transition to states with definite mass involves a unitary matrix U different from unity.

We shall describe the standard parametrization of the matrix U.[6] A unitary matrix $n\otimes n$ is characterized by n^2 real parameters. An orthogonal matrix $n\otimes n$ is characterized by $n(n-1)/2$ parameters. Thus a unitary matrix $n\times n$ is specified by $n(n-1)/2$ angles and $n(n+1)/2$ phases. However, in the case of the matrix of the charged currents, some of the phases can be eliminated by multiplying q_A^i and q_C^j by $e^{i\alpha}$. As a result, there remain $n(n+1)/2-1-(2n-2)=(n^2-3n+2)/2$ independent phases, which characterize the matrix of the charged currents in the Weinberg-Salam model that unifies the $2n$ quarks. In the four-quark model, there are no phases and it is not possible to accommodate CP violation in the interaction of quarks with W bosons. The six-quark model has one CP-odd phase and three Cabibbo-like angles. We write the matrix U in brackets in a form which we shall use in what follows:

$$(\bar{u}\bar{c}\bar{t})\begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3-s_2s_3e^{i\delta} & c_1c_2s_3+s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3+c_2s_3e^{i\delta} & c_1s_2s_3-c_2c_3e^{i\delta}\end{pmatrix}\begin{pmatrix} d \\ s \\ b\end{pmatrix}; \qquad (1)$$

here c_i and s_i are the cosines and sines of the corresponding angles.

It is easy to conclude from the foregoing that the angles θ_1, θ_2, and θ_3 can be assumed to vary within the interval 0 to $\pi/2$, and we shall assume that the phase δ is restricted to the interval $-\pi$ to π.[10] We note here, however, that if the masses of any of the ano- or catho-quarks were equal, then the resulting additional symmetry would make it possible to transform away the phase δ, and the CP violation would disappear.[5]

An analysis of experimental data on β decays of nuclei, semileptonic decays of hyperons, and K_{l3} decay gives $\sin\theta_1=0.23$ and $\sin\theta_3<0.5$.[11]

3. THE TRANSITION $d\bar{s}\to s\bar{d}$ IN THE FREE-QUARK APPROXIMATION

The transition $d\bar{s}\to s\bar{d}$ is determined by the diagrams of Fig. 1.

The calculation of the box diagrams of Fig. 1 is very simple. However, we consider this calculation in detail, anticipating the "dressing" of these diagrams by gluons, which we will encounter in Sec. 4. We henceforth neglect the masses of the light (u, d, s) quarks and the momenta of the external particles. We also neglect m_c^2/m_t^2 in comparison with unity throughout the paper. We give the expressions for the propagators of the W bosons and charged Higgs particles (Ref. 12)[1)] and for

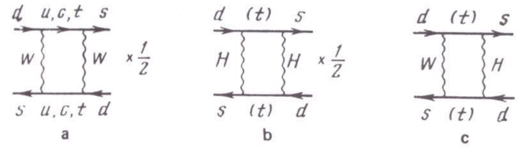
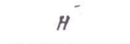


FIG. 1. Diagrams which generate the transition $d\bar{s}\to s\bar{d}$. The fermion propagators in diagrams b and c are labeled (t), since it follows from the text that it is sufficient to consider only the exchange of t quarks in these graphs.

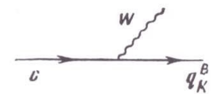
the $W\bar{q}q$ and $H\bar{q}q$ transition vertices in the Weinberg-Salam model in the α gauge, which are needed to calculate the graphs of Fig. 1:

$$D_W=-\left[\frac{g_{\mu\nu}-(k_\mu k_\nu/k^2)}{k^2-M_W^2}+\frac{k_\mu k_\nu/k^2}{k^2\alpha^{-1}-M_W^2}\right] \qquad (A)$$

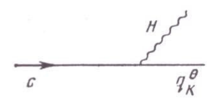


$$D_H=\frac{1}{k^2-M_W^2\alpha}. \qquad (B)$$

In the formulas given below, $q_K^\theta=s_1c_2d+(c_1c_2c_3-s_2s-s_2s_3e^{i\delta})s$; using (1), we can immediately write down the analogous vertices in which the c quark is replaced by the u or t quark:



$$\frac{g}{2\sqrt{2}}W_\mu\bar{q}_K^\theta\gamma_\mu(1+\gamma_5)c+\text{H.c.}, \qquad (C)$$



$$\frac{g}{2\sqrt{2}}\frac{m_c}{M_W}[\bar{q}_K^\theta(1-\gamma_5)cH+\bar{c}(1+\gamma_5)q_K^\theta H^+]. \qquad (D)$$

In what follows, we employ the gauge of 't Hooft and Feynman, in which $\alpha=1$.

Let us calculate the diagram of Fig. 1. The factor $\frac{1}{2}$ indicated in Figs. 1a and 1b occurs when e^{iV} is exposed according to Wick's rules. It is possible for u, c, and t quarks to propagate along the internal fermion lines of Fig. 1a. Using the unitarity of the matrix (1), we subtract $b_i/\hat{p}$ from the propagators of the u, c, and t quarks, where p is the momentum flowing in the loop and b_i is the product of the angles of the matrix (1) at the $W\bar{q}q$ vertices. We are thereby subtracting zero, since $\sum b_i=0$ (see Sec. 2). After this, the propagator of the u quark drops out and we are left with diagrams involving the propagation of c and t quarks, whose propagators take the form $m_{c,t}^2/(p^2-m_{c,t}^2)\hat{p}$. It now remains for us to calculate the Feynman integrals corresponding to three different cases: 1) c quarks propagate along the upper and lower lines; 2) t quarks propagate along the upper and lower lines; 3) a c quark goes along one line, and a t quark along the other. To simplify the expressions containing γ matrices, it is convenient to apply a Fierz transformation. After a Wick rotation, the integral corresponding to the first case takes the form $\int d^4p\, m_c^4M_W^4/(p^3+m_c^2)^2p^2(p^2+M_W^2)^2$. This integral has a power convergence for $p^2=m_c^2$; we contract the W-boson propagators to a point and write down the result: m_c^2. The integral corresponding to the second case has the form $\int d^4p\, m_tM_W/(p^2+m_t^2)^2p^2(p^2+M_W^2)^2$. It is clear that when $m_t^2\ll M_W^2$ this integral is equal to $m_t^2[1+O(m_t^2/M_W^2\ln M_W^2/m_t^2)]$ and, if we are interested in t-quark masses $\leqslant 30$ GeV, we can equate it to m_t^2. However, if we are interested in larger values of m_t, we must evaluate it exactly without contracting the propagators of the W bosons. Thus, in the second case, as in the first, the Feynman integral has a power convergence for $p^2 = m_t^2$. The third integral has the form $\int d^4p m_c^2 m_t^2 M_W^4/(p^2+m_c^2)(p^2+m_t^2)p^2(p^2+M_W^2)^2$. We recall that since $m_t \geqslant 10$ GeV, we have $m_t^2/m_c^2 \gg 1$. The integral is equal to $m_c^2 \ln(m_t^2/m_c^2)$, since in the region $m_t^2/M_W^2 \ll 1$ we are contracting the propagators of the W bosons and the correction behaves as a small power [as $(m_t^2/M_W^2)\ln(M_W^2/m_t^2)$], while in the region $m_t^2/M_W^2 \approx 1$ we can assume that $\ln(m_t^2/m_c^2)$ is large and that the correction is $O(1)$. Thus, in the third case we are dealing with a logarithmic integral.

The final expression for the contribution to $\mathscr{L}_{\Delta S=2}$ from the diagram of Fig. 1a is as follows:

$$\mathscr{L}_{\Delta S=2} = -\frac{g^4}{32\cdot 16\pi^2 M_W^4}[\bar{s}(\gamma_\alpha(1+\gamma_5)d]^2(A_1+A_2+A_3+A_2^{HH}+A_2^H), \quad (2a)$$

$$A_1 = m_c^2 s_1^2 c_2 (c_1c_2c_3 - s_2s_3e^{-i\delta})^2, \quad (2b)$$

$$A_2 = m_t^2 s_1^2 s_2^2 (c_1s_2c_3 + c_2s_3e^{-i\delta})^2 \Big[\frac{M_W^4}{(M_W^2-m_t^2)^2} + \frac{m_t^2 M_W^2}{(M_W^2-m_t^2)^2} + \frac{2m_t^2 M_W^4}{(m_t^2-M_W^2)^3}\ln\frac{M_W^2}{m_t^2}\Big], \quad (2c)$$

$$A_3 = m_c^2 \ln\frac{m_t^2}{m_c^2} 2s_1^2 c_2 s_2 (c_1c_2c_3 - s_2s_3e^{-i\delta})(c_1s_2c_3 + c_2s_3e^{-i\delta}). \quad (2d)$$

Here A_2^{HH} and A_2^H denote the contributions from the diagrams of Figs. 1b and 1c, which are calculated below. We note that the expression in the square brackets in (2c) has no pole at $m_t^2 = M_W^2$ and is equal to $\frac{1}{3}$ at this point.

Let us turn to the calculation of the graphs of Figs. 1b and 1c. These graphs include exchanges of charged Higgs particles, whose emission vertices contain the small factor m_q/M_W in comparison with the W-boson emission vertex. This factor may not be small for a sufficiently heavy t quark. We shall show that it is permissible to consider only the propagation of t quarks along the internal fermion lines in the graphs of Figs. 1b and 1c, and we shall calculate the corresponding contributions. We begin with the graph of Fig. 1b. If a c quark propagates along each of the two fermion lines, it is easy to see that the contribution from the graph of Fig. 1b is suppressed by a factor m_c^2/M_W^2 in comparison with the quantity A_1. But if a t quark propagates along one line and a c quark along the other, then we lose the large logarithm $\ln(m_t^2/m_c^2$ in comparison with A_3 (the logarithm is actually large if $m_t^2 \gg M_W^2$, but if $m_t^2 \ll M_W^2$ there is a power-law suppression m_t^2/M_W^2 in comparison with A_3). Thus we see that it is permissible to consider only the propagation of t quarks along both fermion lines in the graph of Fig. 1b. The Feynman integral which determines A_2^{HH} has the form $\int p^2 d^4p/(p^2+M_W^2)^2(p^2+m_t^2)^2$. We are interested in its value for $m_t^2 \approx M_W^2$, since when $m_t^2 \ll M_W^2$ the contribution from the graphs involving the exchange of Higgs particles is suppressed as m_t^2/M_W^2. When $m_t^2 \approx M_W^2$, the integral has a power behavior and acquires its value mainly from the region $p^2 \approx M_W^2$. The expression for A_2^{HH} is as follows:

$$A_2^{HH} = m_t^2 s_1^2 s_2^2 (c_1s_2c_3 + c_2s_3e^{-i\delta})^2 \Big[\frac{2m_t^4 M_W^2}{4(M_W^2-m_t^2)^3}\ln\frac{m_t^2}{M_W^2} + \frac{m_t^2(M_W^2+m_t^2)}{4(M_W^2-m_t^2)^2}\Big] \quad (2e)$$

The quantity A_2^{HH} has no pole at $m_t^2 = M_W^2$, and the expression in the square brackets is equal to 1/12 at this point (to be compared with $\frac{1}{3}$ in the case of A_2).

We turn now to the graph of Fig. 1c. If a c quark propagates along one or both fermion lines, we have the same suppression in comparison with A_1 and A_3 as in the diagram of Fig. 1b. Let us take t quarks along both lines. The Feynman integral which determines A_2^H has the form $\int d^4p/(p^2+m_t^2)^2(p^2+M_W^2)^2$; we are interested in the region $m_t^2 \approx M_W^2$, in which this integral has a power behavior and acquires its value mainly from the region $p^2 \approx M_W^2$. Thus we obtain

$$A_2^H = m_t^2 s_1^2 s_2^2 (c_1s_2c_3 + c_2s_3e^{-i\delta})^2 \Big[\frac{2m_t^2 M_W^2(m_t^2+M_W^2)}{(M_W^2-m_t^2)^3}\ln\frac{M_W^2}{m_t^2} - \frac{4m_t^2 M_W^2}{(M_W^2-m_t^2)^2}\Big]. \quad (2f)$$

When $m_t^2 = M_W^2$, the expression in the square brackets is equal to $\frac{1}{3}$.

This completes the calculation of $\mathscr{L}_{\Delta S=2}$ in the free-quark approximation. The expression for it is given in Eqs. (2a)–(2f).

4. CALCULATION OF $\mathscr{L}^{\text{eff}}_{\Delta S=2}$ WITH ALLOWANCE FOR THE STRONG INTERACTION

In the preceding section, we obtained an expression for $\mathscr{L}_{\Delta S=2}$ in the free-quark approximation. In this section, we take into account gluon exchanges at small distances and obtain an expression for the effective Lagrangian $\mathscr{L}^{\text{eff}}_{\Delta S=2}$. This program was carried out in the four-quark model in Ref. 3. What is new in comparison with Ref. 3 is the presence of the t quark and the need to dress the diagrams involving exchanges of Higgs particles (Figs. 1b and 1c) with gluons. The scheme of determining the coefficient function of $\mathscr{L}_{\Delta S=2}$ is standrad and reduces to the following two items. We take the bare diagram and dress it with one gluon in all possible ways that give $\ln(p_1^2/p_2^2)$. The problem of operator mixing then becomes completely transparent, and both the set of multiplicatively renormalized operators and the first term of the expansion of $[g^2(p_1^2)/g^2(p_2^2)]^\gamma$ in powers of g^2 can be determined in a trivial manner. The second stage consists in the determination of the index γ from the first term of the expansion found in the first stage, and this also present no difficulty. Our use of the renormalization group differs from the well known applications in that we have a single-loop graph even in the absence of gluon loops. In this case, the calculations are performed in the following order. We fix the momentum p flowing in the loop which contains no gluons and dress the resulting diagram with gluons, finding the anomalous dimensions γ. Then, finally, we calculate the integral with respect to p. This last integral has the form $\int f(p^2)d^4p[g^2(p^2)/g^2(p_1^2)]^\gamma$, and here we must distinguish two cases. If the integral $\int f(p^2)d^4p$ has a power behavior, we obtain the result for it by multiplying $\int f(p^2)d^4p$ by $[g^2(p_2^2)/g^2(p_1^2)]^\gamma$, where p_2 is the momentum for which the integral $\int f(p^2)d^4p$ converges. But if the original integral is logarithmic, we must calculate $\int_{p_1^2}^{p_2^2} dp^2/p^2[g^2(p^2)/g^2(p_1^2)]^{\gamma}$ exactly. A thorough discussion of the need for an accurate calculation of logarithmic integrals is given in Ref. 13 in connection with the decay $K_L \to 2\mu$. This concludes our general remarks, and we turn now to the concrete calculations.

We begin with the dressing of the graph in Fig. 1a. The Landau gauge of the gluon propagators is most convenient for the separation of the logarithms, and we shall use this gauge in what follows. In the Landau gauge, the anomalous dimensions of the fermion propagators are equal to zero, and allowance for the gluon dressings $G_F(q)$ in the leading-logarithm approximation (Fig. 2) reduces to the appearance of a logarithmic factor multiplying m_F:

$$m_F \to m_F\left(1+\frac{b}{4\pi}\frac{g^2(m_F^2)}{4\pi}\ln\frac{q^2}{m_F^2}\right)^{-4/b},$$

where $b = 11 - \frac{2}{3}N_f$, in which N_f is the number of quark flavors present in the logarithmic region ($q^2 \gg m_f^2$); the anomalous dimensions $-4/b$ can be most readily determined by calculating the single-loop graph of Fig. 2b, but the leading logarithms for $G_F(q)$ were selected for the first time in Ref. 14.

Let us consider what gluon exchanges in the graph of Fig. 1a, besides the dressing of the quark propagator which we have already considered, can give a logarithm. Dressing of the vector and axial-vector vertices in the Landau gauge gives no logarithms, nor are logarithms obtained from the "transport" of a gluon across several such vertices along one and the same fermion line, so that the graphs of Figs. 3a and 3b do not contain any logarithm. There is also no logarithm in the graph of Fig. 3c, since it can be reduced to that of Fig. 3d by means of a Fierz transformation. Thus we have obtained a simple and perspicuous rule: graphs which contain gluon exchange along a fermion line, or which can be reduced to such graphs by means of a Fierz transformation, contain no logarithms. There remain two graphs of the type shown in Fig. 4a, and four of the type shown in Fig. 4b, containing logarithms. We begin with the graph of Fig. 4a and consider in what region of integration with respect to the gluon momentum q it is logarithmic. The two fermion propagators and the gluon propagator ensure a logarithmic behavior in the region $\mu^2 < q^2 < p^2$, where for the lower limit we take μ^2 such that $g^2(\mu^2)/4\pi = 1$, hoping that an arbitrariness in the choice of μ^2 does not result in a large error; the upper limit is the momentum at which the third quark propagator comes into play and cuts off the logarithmic integral. It is easy to understand that this occurs when $q^2 \approx p^2$. We must now calculate the graphs of Fig. 4a at fixed p, see that operator mixing does not occur—there is only a single operator $\bar{s}_i\gamma_\alpha(1+\gamma_5) \times d_i\bar{s}_j\gamma_\alpha(1+\gamma_5)d_j$ (i and j are color indices), and collect the leading logarithms. The following two identities for the γ and t matrices are helpful in the calculation:

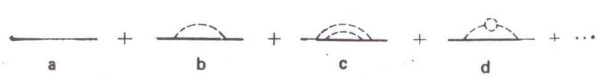


FIG. 2. Dressing of the quark propagator.

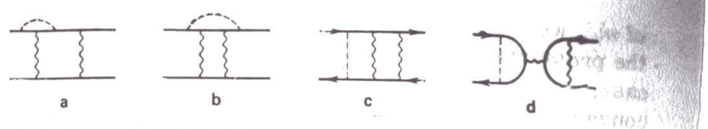


FIG. 3. Graphs which do not contain logarithms in the Landau gauge.

$$\gamma_\alpha\gamma_\beta\gamma_\gamma = g_{\alpha\beta}\gamma_\gamma + g_{\beta\gamma}\gamma_\alpha - g_{\alpha\gamma}\gamma_\beta - i\varepsilon_{\alpha\beta\gamma\delta}\gamma_5\gamma_\delta,$$

$$t_{ij}^a t_{mn}^a = {}^1/_2\,\delta_{in}\delta_{jm} - {}^1/_6\,\delta_{ij}\delta_{mn}$$

(the gluon emission vertex has the form $gA_\mu^a\bar{q}\gamma_\mu t^a q$). The calculation leads to a factor $[g^2(\mu^2)/g^2(p^2)]^{-2/b}$ in the integrand of the integral with respect to d^4p, where $b = 11 - \frac{2}{3}N_f$. We shall consider here the value of N_f. It is equal to the number of quark flavors which can be assumed to be massless in the momentum region that gives the main contribution to the logarithms of the renormalization group, and b can vary between 9 ($N_f = 3$) and 7 ($N_f = 6$). Actually, in the cases under consideration p^2 is always less than M_W^2, and the b and t quarks do not become logarithmic [i.e., $\ln(M_W^2/m_q^2)$ does not become large in comparison with unity]; thus, we shall encounter the values $b = 9$ ($N_f = 3$) and 25/3 ($N_f = 4$). For $p^2 \approx M_W^2$, it is perhaps possible to assume that the b quark ($m_b = 5$ GeV) is also logarithmic. However, the distinction between the numerical values of b for $N_f = 4$ (25/3) and $N_f = 5$ (25/3) is very small and does not lead to appreciable numerical differences.

We turn now to the graphs of Fig. 4b. To investigate their dressing by gluons, we cut them along the two internal fermion lines. We then see that each of the halves of the diagram is an analog of the operator whose renormalization was studied in Ref. 15 in connection with the $\Delta T = \frac{1}{2}$ rule in nonleptonic decays with $\Delta S = 1$. Let us see what region gives the main contribution to the logarithm in the graphs of Fig. 4b. The propagators of the gluon and the external (d or s) quark give three powers in the denominator, and the missing fourth power must come from the propagator of the internal quark. The upper limit of the logarithmic integral is cut off at $q^2 = M_W^2$ by the propagator of the W boson. By analyzing the propagator of the internal quark, it is easy to understand that the region of logarithmic behavior is $M_W^2 > q^2 > p^2$. Thus, using the result of Ref. 15, we see that each of the halves of the diagram of Fig. 4b is renormalized as follows:

$$\bar{s}_i\gamma_\alpha(1+\gamma_5)q_i\bar{q}_k\gamma_\alpha(1+\gamma_5)d_k \to \left[\frac{g^2(p^2)}{g^2(M_W^2)}\right]^{-2/b} O_+ + \left[\frac{g^2(p^2)}{g^2(M_W^2)}\right]^{4/b} O_-,$$

$$O_\pm = {}^1/_2[\bar{q}_i\gamma_\alpha(1+\gamma_5)d_i\bar{s}_k\gamma_\alpha(1+\gamma_5)q_k \pm \bar{q}_i\gamma_\alpha(1+\gamma_5)q_i\bar{s}_k\gamma_\alpha(1+\gamma_5)d_k],$$

where q denotes a u, c, or t quark. The formulas gives above differ from the analogous ones of Ref. 15 in that μ^2 is replaced by p^2. We must now link these

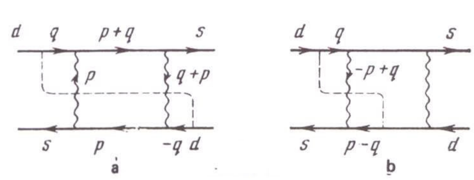


FIG. 4. Graphs which give a logarithmic renormalization.

halves, using Wick's rules. The final result for the factor in the integrand of the diagram of Fig. 4b is

$$\frac{1}{2}\left[\frac{g^2(p^2)}{g^2(M_W^2)}\right]^{8/b}-\left[\frac{g^2(p^2)}{g^2(M_W^2)}\right]^{2/b}+\frac{3}{2}\left[\frac{g^2(p^2)}{g^2(M_W^2)}\right]^{-4/b}$$

(this formula, with p^2 replaced by m_c^2, is given in Ref. 3). The factors corresponding to the diagrams of Figs. 4a and 4b were obtained in Ref. 3, but since the four-quark model was considered in Ref. 3, the external integrals were the same and converged according to a power law to m_c^2 [which corresponds to A_1 in (2b)], and also p^2 was everywhere replaced by m_c^2. Here we have three integrals (A_1, A_2, and A_3), and we shall now write out the factors which renormalize them. Let us denote them by η_1, η_2, and η_3, so that the contribution to $\mathscr{L}^{\text{eff}}_{\Delta S=2}$ from the diagrams of Fig. 1a dressed with gluons has the form

$$\eta_1 A_1+\eta_2 A_2+\eta_3 A_3. \tag{3}$$

Since the integrals with respect to d^4p for A_1 and A_2 have a power behavior, we can immediately write down the results for η_1 and η_2:

$$\eta_1=\left[\frac{g^2(\mu^2)}{g^2(m_c^2)}\right]^{-2/9}\left\{\frac{1}{2}\left[\frac{g^2(m_c^2)}{g^2(M_W^2)}\right]^{24/25}-\left[\frac{g^2(m_c^2)}{g^2(M_W^2)}\right]^{6/25}+\frac{3}{2}\left[\frac{g^2(m_c^2)}{g^2(M_W^2)}\right]^{-12/25}\right\}, \tag{4}$$

$$\eta_2=\left[\frac{g^2(\mu^2)}{g^2(m_t^2)}\right]^{-2/9}\left\{\frac{1}{2}\left[\frac{g^2(m_t^2)}{g^2(M_W^2)}\right]^{24/25}-\left[\frac{g^2(m_t^2)}{g^2(M_W^2)}\right]^{6/25}+\frac{3}{2}\left[\frac{g^2(m_t^2)}{g^2(M_W^2)}\right]^{-12/25}\right\}. \tag{5}$$

In the calculation of η_3, we must remember that the integral which determines A_3 is logarithmic and that the result for A_3 contains $m_c^2\ln(m_t^2/m_c^2)$. Therefore, to calculate η_3, we must take into account the renormalization of m_c and evaluate the integrals with respect to $(dp^2/p^2)(\ln p^2)^\alpha$ exactly without removing $(\ln p_1^2)^\alpha$ from the integral signs, where p_1 is one of the limits of integration. Doing this, we obtain

$$\eta_3=\frac{1}{\ln(m_t^2/m_c^2)}\left[\frac{g^2(\mu^2)}{g^2(m_c^2)}\right]^{-2/9}\frac{3\cdot 16\pi^2}{25g^2(m_c^2)}\left\{\frac{1}{2}\left[\frac{g^2(m_c^2)}{g^2(M_W^2)}\right]^{24/25}\right.$$
$$\times\frac{225}{257}\left[1-\left(\frac{g^2(m_t^2)}{g^2(m_c^2)}\right)^{257/225}\right]-\left[\frac{g^2(m_c^2)}{g^2(M_W^2)}\right]^{6/25}\frac{45}{19}\left[1-\left(\frac{g^2(m_t^2)}{g^2(m_c^2)}\right)^{19/45}\right]$$
$$\left.+\frac{3}{2}\left[\frac{g^2(m_c^2)}{g^2(M_W^2)}\right]^{-12/25}\frac{225}{67}\left[\left(\frac{g^2(m_t^2)}{g^2(m_c^2)}\right)^{-67/225}-1\right]\right\}. \tag{6}$$

We stress that η_1, η_2, and η_3 reduce to unity when the strong interaction is switched off. For η_1 and η_2 this is immediately obvious, and in the case of η_3 we must expand $g^2(m_t^2)/g^2(m_c^2)$ in the curly brackets in (6), retaining the term $\sim g^2(m_c^2)$.

It remains for us to add gluon dressings to the graphs of Figs. 1b and 1c containing exchanges of Higgs particles. We begin with the graph of Fig. 1b. By direct calculation, it is easy to see that the graphs of Figs. 5a and 5b contain no logarithms. The $\bar{q}qH$ vertex, when dressed with a gluon (Fig. 6a), contains a logarithm which is not cut off in any way and which formally goes to infinity, and this compels us to pay special attention to this graph. If H were not a Higgs particle, but an ordinary scalar particle, we would have to fix the $\bar{q}qH$ vertex at some point, and this would create a prescription for the calculation of the gluon loop in Fig. 6a. However, H is the Higgs boson of the rigid Weinberg–Salam scheme, in which introduction of the counterterm $\delta c\bar{s}(1-\gamma_5)tH$ is forbidden by isotopic invariance. The only counterterm which can be introduced in the interaction of Higgs particles with quarks is the counterterm $\delta f\bar{\psi}_L t_R H$ of quark-mass renormalization by the strong interaction, where the value of δf is uniquely fixed by the requirement that the renormalized quark mass should be equal to the observed value. It is remarkable that this same quantity δf removes the divergent logarithm in the interaction of the physical neutral and unphysical charged Higgs particles with the $\bar{q}q$ pair. Thus the graph of Fig. 6a contains $g^2\ln(M_W^2/m_t^2)$ and, since Higgs exchanges are important only for $m_t^2\approx M_W^2$, it can be neglected. Consequently, exactly as in the case of the exchange of two W bosons, we are left with the graphs of Figs. 6b and 6c. The graph of Fig. 6c is not important when $\ln(M_W^2/m_t^2)\approx 1$; selection of the leading logarithms in the graphs of the type shown in Fig. 6b gives

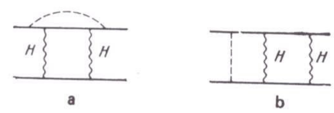


FIG. 5. Graphs which do not give a logarithm in the Landau gauge.

$$\eta_{HH}=\left[\frac{g^2(\mu^2)}{g^2(M_W^2)}\right]^{-2/9}. \tag{7}$$

If the graph of Fig. 1c is dressed by exactly the same procedure as that which we have just carried out, we find

$$\eta_H=\eta_{HH}. \tag{8}$$

To conclude this section, we give the formula for $\mathscr{L}^{\text{eff}}_{\Delta S=2}$:

$$\mathscr{L}^{\text{eff}}_{\Delta S=2}=-\frac{g^4}{32\cdot 16\pi^2 M_W^4}[\bar{s}\gamma_\alpha(1+\gamma_5)d]^2 \times[\eta_1 A_1+\eta_2 A_2+\eta_3 A_3+\eta_{HH}A^{HH}+\eta_H A_2^H]; \tag{9}$$

the expressions for η_1–η_H are given in (4)–(8), and those for A_1–A_2^H are given in (2b)–(2f). This completes the calculation of $\mathscr{L}^{\text{eff}}_{\Delta S=2}$. It now remains for us to find its matrix element between the K^0 and $\bar{K}^0$ states, after which we can immediately obtain formulas for Δm_{LS} and for the parameter ε of CP violation.

5. CALCULATION OF $\langle K^0|\mathscr{L}^{\text{eff}}_{\Delta S=2}|\bar{K}^0\rangle$, Δm_{LS}, AND ε IN THE SIX-QUARK MODEL

In the preceding sections, we have calculated $\mathscr{L}^{\text{eff}}_{\Delta S=2}$. To calculate Δm_{LS} and ε, we must now find the matrix element $\langle K^0|\bar{s}\gamma_\alpha(1+\gamma_5)d\bar{s}\gamma_\alpha(1+\gamma_5)d|\bar{K}^0\rangle$.[2] To do this, we isolate the product of the currents in $\mathscr{L}_{\Delta S=2}$ and

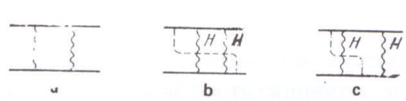


FIG. 6. Graphs which produce a logarithmic renormalization.

saturate it with all possible intermediate states:

$$\langle K^0|\bar{s}_i\gamma_\alpha(1+\gamma_5)d_i\bar{s}_k\gamma_\alpha(1+\gamma_5)d_k|\bar{K}^0\rangle$$
$$={}^8/_3\sum_n\langle K^0|\bar{s}_i\gamma_\alpha(1+\gamma_5)d_i|n\rangle\langle n|\bar{s}_k\gamma_\alpha(1+\gamma_5)d_k|\bar{K}^0\rangle, \qquad (10)$$

where the factor 8/3 occurs because of the two types of couplings of the color indices.

In the sum over intermediate states, we confine ourselves to the two lowest states, the vacuum state and the single-pion state.[3] As we shall see, the contribution of the single-pion state turns out to be negligibly small in comparison with that of the vacuum state, and this will serve as the basis for the retention in the sum (10) of only the first term—the vacuum insertion.

To calculate the contribution to the matrix element $\langle K^0|\mathscr{L}|\bar{K}^0\rangle$ from the vacuum intermediate state, we make use of the relation $\langle K^0|\bar{s}_i\gamma_\alpha(1+\gamma_5)d_i|0\rangle = if_K P^\alpha$. We then obtain

$$\langle K^0|\bar{s}\gamma_\alpha(1+\gamma_5)d|0\rangle\langle 0|\bar{s}\gamma_\alpha(1+\gamma_5)d|\bar{K}^0\rangle=-f_K^2m_K^2.$$

Let us estimate the contribution from the single-pion intermediate state. It is determined by the two graphs of Fig. 7. Using the form $\langle K^0|\bar{s}\gamma_\alpha(1+\gamma_5)d|\pi^0\rangle=(1/\sqrt{2})(p_\pi+p_K)_\alpha$ for the matrix element of the K–π transition (we have neglected the contribution of the form factor f_- and replaced $f_+(q^2)$ by its value at $q^2=0$ in the $SU(3)$ limit), we obtain

$$M_a+M_b=\frac{1}{2}\int\frac{d^3q}{(2\pi)^3 2q_0}(p+q)^2+\frac{1}{2}\int\frac{d^3q}{(2\pi)^3 2(q_0+2m_K)}(p-q)^2.$$

We truncate the integral with respect to the energy of the pion at $q_0=\Lambda$. We then obtain

$$\langle K^0|\bar{s}\gamma_\alpha(1+\gamma_5)d|\pi^0\rangle\langle\pi^0|\bar{s}\gamma_\alpha(1+\gamma_5)d|\bar{K}^0\rangle$$
$$=\frac{m_K^2}{4\pi^2}\left[\frac{3}{2}\Lambda^2-5m_K\Lambda+10m_K^2\ln\frac{\Lambda+2m_K}{2m_K}\right].$$

It is now necessary to understand at what value of Λ the integral should be truncated. This must be a value of Λ at which the strong interaction becomes weak and beyond which hadron–quark duality sets in. If we truncate the integral above this value, we are again including the contribution which is already taken into account by the quark diagrams of Figs. 1–6. A phenomenological analysis of the vacuum average of the four-fermion operator constructed from the u and d quark fields in Ref. 18 shows that Λ must be set equal to ≈200 MeV. Even if we overestimate the contribution of the pion insertion and put $\Lambda=500$ MeV, this contribution amounts to 10% of the vacuum contribution.

A calculation of $\langle K^0|[\bar{s}\gamma_\alpha(1+\gamma_5)d]^2|\bar{K}^0\rangle$ in the MIT bag model which was carried out in Ref. 8 led to the appearance of a numerical factor 0.42 in front of the result for the vacuum insertion. The foregoing discussion shows that in a correct calculation this factor must be close to unity, and we shall now indicate what might account for the inapplicability of the bag model in the calculation of the matrix element in question. In the first place, the K^0 meson is a Goldstone particle and its mass should reduce to zero if the masses of the s and d quarks are equal to zero. The bag model does not satisfy this requirement and cannot correctly describe the physics of pions and kaons. In particular, our discussion shows that the matrix element $\langle K^0|[\bar{s}\gamma_\alpha(1+\gamma_5)d]^2|\bar{K}^0\rangle$ is almost completely determined by the vacuum insertion and, consequently, by the value of f_K; on the other hand, the bag model does not reproduce the constants f_π and f_K. Finally, in the calculation performed in Ref. 8 the vacuum insertion was replaced by the insertion of an empty bag—a state which is absent in the spectrum of physical particles, and this is also unsatisfactory.

Thus, the combination of arguments presented in this section shows that the following equality holds with accuracy exceeding 10%:

$$\langle K^0|[\bar{s}\gamma_\alpha(1+\gamma_5)d]^2|\bar{K}^0\rangle=-{}^8/_3 m_K^2 f_K^2. \qquad (11)$$

We are now ready to write down the formulas for Δm_{LS}, we can assume that $K_S=(K+\bar{K})/\sqrt{2}$ and $K_L=(K-\bar{K})/\sqrt{2}$, in which case

$$-2m_{K^0}(m_{K_L}-m_{K_S})=\langle K^0|H_{\Delta S=2}|\bar{K}^0\rangle+\langle\bar{K}^0|H_{\Delta S=2}|K^0\rangle, \qquad (12)$$

where $H_{\Delta S=2}=-\mathscr{L}_{\Delta S=2}$ and the factor $2m_K 0$ on the left-hand side of (12) is due to the normalization of the kaon field: $\langle K^0|H|K^0\rangle=2m_K^2 0$. We rewrite Eq. (12) once again:

$$\Delta m_{LS}=\frac{4}{3\pi}\frac{\Gamma_{K\to\mu\nu}}{m_\mu^2 s_1^2 c_3^2}\mathrm{Re}[A_1\eta_1+A_2\eta_2+A_3\eta_3+A_2^{HH}\eta_{HH}+A_2^H\eta_H], \qquad (13)$$

where A_1–A_2^H and η_1–η_H are given in Eqs. (2b)–(2f) and (4)–(8). Equation (13) expresses the mass difference of the K_L and K_S mesons, which is known from experiment, in terms of the parameters of the six-quark model, many of which are still unknown at the present time. In this connection, we represent (13) in the form

$$\frac{1}{s_1^2c_3^2}\mathrm{Re}[A_1\eta_1+\ldots+A_2^H\eta_H]=2{,}7\,\mathrm{GeV}^2. \qquad (14)$$

In the four-quark model, $A_1=m_c^2s_1^2$, $A_2=\cdots A_2^H=0$, $c_3^2=1$, and $\eta_1=0.6$ (see below). Since $m_c=1.3$ GeV,[19] the necessity of the six-quark generalization of the four-quark model is apparent.

Turning now to the parameter ε of CP violation, we write the general expression for ε in the standard notation:

$$\varepsilon=(m_{12}-i\gamma_{12})/(m_1-m_2+i\gamma_1-i\gamma_2).$$

It is known from experiment that $\gamma_{12}\ll m_{12}$. In our approximation, $\gamma_{12}=0$. The experimental value of the phase ε is approximately 45°. Using this, we readily obtain the following expression for the modulus of ε:

$$|\varepsilon|=\frac{\langle K^0|H|\bar{K}^0\rangle-\langle\bar{K}^0|H|K^0\rangle}{2\sqrt{2}[\langle K^0|H|\bar{K}^0\rangle+\langle\bar{K}^0|H|K^0\rangle]}=\frac{\mathrm{Im}[A_1\eta_1+\ldots+A_2^H\eta_H]}{2\sqrt{2}\,\mathrm{Re}[A_1\eta_1+\ldots+A_2^H\eta_H]}. \qquad (15)$$

The experimental value is $|\varepsilon|=2.3\times10^{-3}$.

Thus, we have obtained formulas [(13) and (15)] which express the mass difference of the K_L and K_S mesons and the parameter ε of CP violation in terms of the

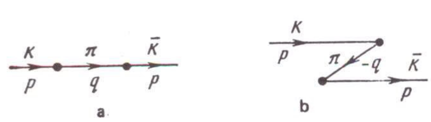


FIG. 7. Graphs which determine the contribution of the single-pion intermediate state to the matrix element $\langle K^0|\mathscr{L}^{\mathrm{eff}}_{\Delta S=2}|\bar{K}^0\rangle$.

parameters of the six-quark model. We now make some numerical estimates.

6. NUMERICAL ESTIMATES

Of the parameters of the six-quark model, we do not know the phase δ and the angle θ_2. For the angle θ_3 we know that $\sin\theta_3 < 0.5$, and for the mass of the t quark we know that $m_t > 15$ GeV. In this situation, we proceed as follows. We take two values of θ_3, corresponding to $\sin\theta_3 = 0.1$ and $\sin\theta_3 = 0.3$, and three values of the t-quark mass, $m_t = 15$, 30, and 60 GeV. For these six sets of parameters, we find δ and θ_2 satisfying Eqs. (13) and (15), after which we write out the resulting Kobayashi–Maskawa matrices.

We begin the estimates with an examination of the role of the Higgs exchanges in Figs. 1b and 1c. In Table I we give the numbers appearing in the square brackets in the formulas for A_2 [Eq. (2c)] and $A_2^{HH} + A_2^{H}$ [Eqs. (2e) and (2f)]; this table shows how the relative contribution of the Higgs exchanges varies with increasing m_t. We see that with increasing m_t the relative contribution of the Higgs exchanges rises, and their role leads to the maintenance of the sum of the quantities in the square brackets in (2c), (2e), and (2f) at an approximately constant level: 0.98 at $m_t = 15$ GeV, and 0.83 at $m_t = 60$ GeV.

We turn now to the estimates of the renormalization factors η. The values of the strong-interaction constants which we require are as follows ($M_W = 84$ GeV and $m_c = 1.3$ GeV):

$$\frac{g^2(\mu^2)}{4\pi} = 1, \quad \frac{g^2(m_c^2)}{4\pi} = 0.2, \quad \frac{g^2(M_W^2)}{4\pi} = 0.1; \quad \frac{g^2(m_t^2)}{4\pi} = 0.12;\ 0.11;\ 0.1$$

for $m_t = 15$, 30, and 60 GeV, respectively, and $\mu = 80$ MeV. [If we take $\mu = 160$ MeV, the strong-interaction constants which we use change to $g^2(m_c^2)/4\pi = 0.25$, $g^2(M_W^2)/4\pi = 0.11$, and $g^2(m_t^2)/4\pi = 0.14$, 0.13, and 0.11, respectively. Thus we see that there is little change in our results if the value of μ is doubled.] We find $\eta_1 = 0.6$ and $\eta_H = \eta_{HH} = 0.6$; the values of η_2 and η_3 are compiled in Table II. We see that η_2 and η_3 change very little as m_t varies from 15 to 60 GeV.

We are now ready to calculate θ_2 and δ from Eqs. (14) and (15) for each pair (m_t, θ_3) and, by substituting them in the mixing matrix, to obtain this matrix.

The results of the calculations are compiled in Table III.

We remark that the existence of two solutions of the system of equations (14) and (15), corresponding to

TABLE I. Growth of the contributions of Higgs exchanges in comparison with the contribution of W-boson exchanges with increasing m_t.

m_t, GeV	Quantity in the square brackets in Eq. (2c)	Sum of the quantities in the square brackets in Eqs. (2e) and (2f)
15	0.86	0.12
30	0.69	0.25
60	0.45	0.38

TABLE II. Values of the gluon renormalization factors η_2 and η_3 for various masses of the t quark.

m_t, GeV	η_2	η_3
15	0.58	0.46
30	0.59	0.42
60	0.60	0.4

$\cos\delta = \pm 1$, was noted in Ref. 4.

Let us compare our results with those of Ref. 4. The paper of Barger *et al.* differs from ours in that it does not treat gluon exchanges, i. e., all the quantities η are equal to unity in that paper. Barger *et al.* estimate the matrix element in two ways: by means of the vacuum insertion, and according to the bag model. As we pointed out in Sec. 5, the result of the bag model reduces to the appearance of a factor 0.42 in front of the vacuum insertion. Numerically, this factor is virtually the same as the renormalization factors η which we have taken into account. In this connection, our values of s_2 and $\sin\delta$ are practically identical to those obtained by Barger *et al.* with the value $B = 0.42$. The numerical similarity of the results obtained by means of the vacuum insertion and allowance for gluon exchanges in the effective Lagrangian on the one hand, and by calculation of the matrix element according to the bag model with the "bare" quark Lagrangian on the other hand, is purely fortuitous. It by no means implies that the calculation of the matrix element according to the bag model has automatically taken into account the effects of gluon exchanges at small distances. On the contrary, the calculation of the matrix element according to the bag model is evidently an illegitimate procedure (see Sec. 5).

Shrock *et al.* estimated the matrix element according to the bag model; they did not take into account exchanges of gluons or Higgs particles. For the neglect of Higgs exchanges to be legitimate, they are obliged to consider comparatively light t quarks ($m_t = 15$ and 30 GeV). In the analysis of their formulas, Shrock *et al.* introduced a factor $\xi = 2$, by means of which they attempted to take into account possible corrections to the theoretical formulas in the following way:

$$\frac{1}{\xi} < R_\varepsilon, \quad R_{\Delta m} < \xi, \text{ where } R_{\Delta m} \equiv (\Delta m)_{\text{theor}}/(\Delta m)_{\text{exp}}, \quad R_\varepsilon \equiv |\varepsilon|_{\text{theor}}/|\varepsilon|_{\text{exp}}.$$

As a result, each value of s_3 and m_t corresponds in their analysis not to two pairs of values of s_2 and δ, as in our case, but to a whole range of values. Our points

TABLE III. Values of $\sin\theta_2$ and $\sin\delta$ for t-quark masses 15, 30, and 60 GeV and for $\sin\theta_3 = 0.1$ and 0.3.

$\sin\theta_3$; $\cos\delta$	$m_t = 15$ GeV		$m_t = 30$ GeV		$m_t = 60$ GeV	
	$\sin\theta_2$	$\sin\delta$	$\sin\theta_2$	$\sin\delta$	$\sin\theta_2$	$\sin\delta$
0.1; 1	0.28	0.024	0.19	0.017	0.13	0.013
0.1; −1	0.37	0.018	0.29	0.011	0.22	0.007
0.3; 1	0.20	0.010	0.12	0.008	0.07	0.007
0.3; −1	0.49	0.005	0.42	0.003	0.37	0.001

lie in the middle of the ranges of values of Shrock *et al.*

The values of the quark mixing angles which we have found enable us to improve the precision of the results of Ref. 7 for $|\varepsilon'/\varepsilon|$ (see the Introduction). The expression for $|\varepsilon'/\varepsilon|$ obtained in Ref. 7 contains the product $s_2s_3\sin\delta$. The following bounds on this quantity were used in Ref. 7:

$$0.65\cdot10^{-3}<s_2s_3\sin\delta<0.9\cdot10^{-3},\quad m_t=15\ \text{GeV},$$
$$0.13\cdot10^{-3}<s_2s_3\sin\delta<0.47\cdot10^{-3},\quad m_t=75\ \text{GeV}.$$

Here we obtain

$$0.60\cdot10^{-3}<s_2s_3\sin\delta<0.73\cdot10^{-3},\quad m_t=15\ \text{GeV},$$
$$0.11\cdot10^{-3}<s_2s_3\sin\delta<0.17\cdot10^{-3},\quad m_t=60\ \text{GeV}.$$

For $|\varepsilon'/\varepsilon|$ we then obtain

$$0.92\cdot10^{-3}<|\varepsilon'/\varepsilon|<1.1\cdot10^{-3},\quad m_t=15\ \text{GeV},$$
$$0.25\cdot10^{-3}<|\varepsilon'/\varepsilon|<0.40\cdot10^{-3},\quad m_t=60\ \text{GeV},$$

whereas in Ref. 7

$$1.0\cdot10^{-3}<|\varepsilon'/\varepsilon|<1.4\cdot10^{-3},\quad m_t=15\ \text{GeV},$$
$$0.3\cdot10^{-3}<|\varepsilon'/\varepsilon|<1.1\cdot10^{-3},\quad m_t=75\ \text{GeV}.$$

The present experimental bound is $|\varepsilon'/\varepsilon|<0.017$.[20] The ratio $|\varepsilon'/\varepsilon|$ was calculated in Ref. 10, where the value $\varepsilon'\approx(1/50)\varepsilon$ was obtained. This value is an overestimate, since the calculations of Ref. 10 do not allow for the importance of small distances in the loops with heavy quarks (see Ref. 7).

7. CONCLUSIONS

For the Kobayashi–Maskawa variant of the Weinberg–Salam model, we have obtained $\mathscr{L}^{eff}_{\Delta S=2}$ [Eq. (9)], in the calculation of which allowance was made for gluon exchanges at small distances (numerically, this reduces to the appearance of a factor ≈0.6 in $\mathscr{L}^{eff}_{\Delta S=2}$) and exchanges of charged Higgs particles, which are important for $m_t\approx M_W$. The resulting Lagrangian is used to find expressions for Δm_{LS} and ε in terms of the parameters of the six-quark model, and the result of the vacuum insertion is used in the calculation of the matrix element. Values are found for the quark mixing angles when $m_t=15$, 30, and 60 GeV and $s_3=0.1$ and 0.3; the numerical differences between these results and those found previously in Ref. 4 were discussed in Sec. 6.

We have used the values obtained for the mixing angles to improve the accuracy of the quantity $|\varepsilon'/\varepsilon|$, which had been calculated previously in this model and which exhibits a discrepancy between the predictions of the Kobayashi–Maskawa model and the superweak model. Further information on the quark mixing angles will be obtained from the study of the weak decays of particles containing c, b, and t quarks.

Recently, the following experimental result was obtained[21]:

$$\frac{\Gamma(D^0\to\pi^+\pi^-)}{\Gamma(D^0\to K^-\pi^+)}=(3.3\pm1.4)\,\%.$$

In the six-quark model,

$$B=\frac{\Gamma(D^0\to\pi^+\pi^-)}{\Gamma(D^0\to K^-\pi^+)}=\frac{|s_1c_2|^2}{|c_1c_2c_3-s_2s_3e^{-2\delta}|^2}=5.3\%\,\frac{c_2^2}{|c_1c_2c_3-s_2s_3e^{-i\delta}|^2}.$$

Taking the combination of mixing angles with $\cos\delta=-1$ (Ref. 22) and the maximum value of s_2s_3 ($m_t=15$ GeV, $s_3=0.3$, and $s_2=0.49$), we obtain $B=4.5\%$. The size of the error in the experimental value of $\Gamma(D^0\to\pi^+\pi^-)/\Gamma(D^0\to K^-\pi^+)$ and the uncertainty associated with the violation of $SU(3)$ symmetry almost certainly prevent us from finally settling on this combination of mixing angles at the present time, but it is clear that experimental results on the decays of the new particles will soon make it possible to determine the Kobayashi–Maskawa matrix which is realized in nature.

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1) We emphasize that the charged Higgs particles are not physical.

2) The way of looking at the calculation of the matrix element described here arose as a result of discussions with A. I. Vaĭnshteĭn, M. B. Voloshin, I. Yu. Kobzarev, E. P. Shabalin, and M. A. Shifman.

3) Our treatment of the matrix element $\langle K^0|\mathscr{L}|\bar{K}^0\rangle$ is similar to that of Ref. 16. A qualitative treatment of this matrix element had been given earlier; see Ref. 17.

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