The Coulomb law and atomic levels in a superstrong B

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based on M.V.
Pisma v ZhETF 92 (2010) 22;
B.Machet, M.V.
Phys.Rev. D 83 (2011) 025022;
S.Godunov, B.Machet, M.V.
Phys.Rev. D 85 (2012) 044058.

2 problems

1. Energy of the hydrogen ground level in external B in the limit $B \longrightarrow \infty$;

2. The value of Z_{cr} at $B >> B_0$,
where $B_0 \equiv m_e^2/e$ is the Schwinger magnetic field.

The Coulomb potential in $d = 1$

$$\Phi(\bar{k}) \equiv A_0(\bar{k}) = \frac{4\pi g}{\bar{k}^2} \; ; \;\; \mathbf{\Phi} \equiv \mathbf{A}_0 = D_{00} + D_{00}\Pi_{00}D_{00} + \ldots$$

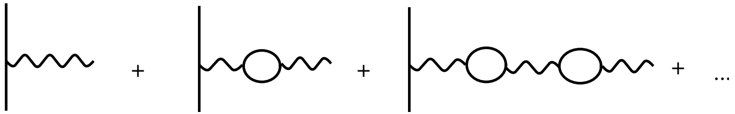


Fig 1. *Modification of the Coulomb potential due to the dressing of the photon propagator.*

Summing the series we get:

$$\mathbf{\Phi}(k) = -\frac{4\pi g}{k^2 + \Pi(k^2)} \; , \;\; \Pi_{\mu\nu} \equiv \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \Pi(k^2), \;\; \Pi \equiv -4g^2 P$$

$$
\begin{aligned}
\mathbf{\Phi} &= 4\pi g \int\limits_{-\infty}^{\infty} \frac{e^{ik_\parallel z} dk_\parallel/2\pi}{k_\parallel^2 + 4g^2(k_\parallel^2/2m^2)/(3 + k_\parallel^2/2m^2)} = \\
&= \frac{4\pi g}{1+2g^2/3m^2} \int\limits_{-\infty}^{\infty} \left[\frac{1}{k_\parallel^2} + \frac{2g^2/3m^2}{k_\parallel^2 + 6m^2 + 4g^2}\right] e^{ik_\parallel z} \frac{dk_\parallel}{2\pi} = \\
&= \frac{4\pi g}{1+2g^2/3m^2} \left[-\frac{1}{2}|z| + \frac{g^2/3m^2}{\sqrt{6m^2+4g^2}} \exp(-\sqrt{6m^2+4g^2}|z|)\right]
\end{aligned}
$$

In the case of heavy fermions ($m \gg g$) the potential is given by the tree level expression; the corrections are suppressed as g^2/m^2.

In the case of light fermions ($m \ll g$):

$$\mathbf{\Phi}(z)\Big|_{\ m \ll g} = \begin{cases} \pi e^{-2g|z|} & , \quad z \ll \frac{1}{g}\ln\left(\frac{g}{m}\right) \\ -2\pi g\left(\frac{3m^2}{2g^2}\right)|z| & , \quad z \gg \frac{1}{g}\ln\left(\frac{g}{m}\right) \end{cases} .$$

$m = 0$ - Schwinger model; photon get mass.

Light fermions - continuous transition from $m > g$ to $m = 0$.

The Coulomb potential in $d = 3$ in strong B 1

$$B >> B_0 \equiv m_e^2/e$$

$$\mathbf{\Phi} = \frac{4\pi e}{(k_{\|}^2 + k_{\perp}^2)\left(1 - \frac{\alpha}{3\pi}\ln\left(\frac{eB}{m^2}\right)\right) + \frac{2e^3 B}{\pi}\exp\left(-\frac{k_{\perp}^2}{2eB}\right) P\left(\frac{k_{\|}^2}{4m^2}\right)} .$$

P the same, as in $d = 1$!

The Coulomb potential in $d = 3$ in strong B 2

$$\Phi(z) =$$

$$= 4\pi e \int \frac{e^{ik_\parallel z} dk_\parallel d^2 k_\perp/(2\pi)^3}{k_\parallel^2 + k_\perp^2 + \frac{2e^3 B}{\pi}\exp(-k_\perp^2/(2eB))(k_\parallel^2/2m_e^2)/(3 + k_\parallel^2/2m_e^2)} \ ,$$

$$\Phi(z) = \frac{e}{|z|}\left[1 - e^{-\sqrt{6m_e^2}|z|} + e^{-\sqrt{(2/\pi)e^3 B + 6m_e^2}|z|}\right] \ .$$

For magnetic fields $B \ll 3\pi m^2/e^3$ the potential is the Coulomb up to small power suppressed terms:

$$\Phi(z)\Bigg|_{\ e^3 B \ll m_e^2} = \frac{e}{|z|}\left[1 + O\left(\frac{e^3 B}{m_e^2}\right)\right]$$

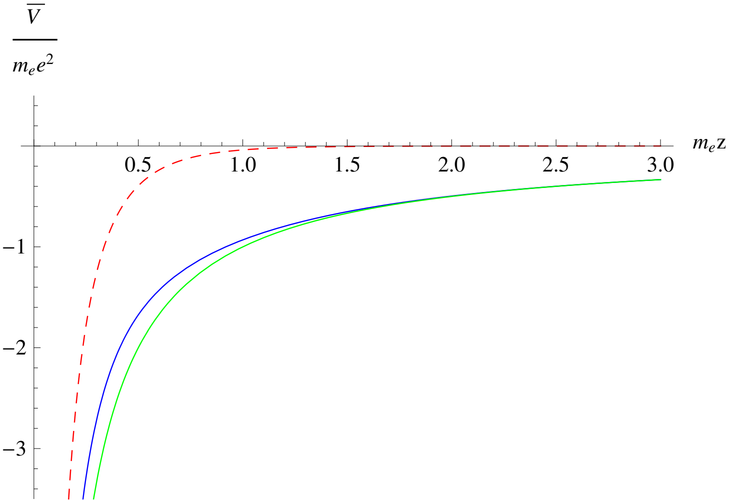
in full accordance with the $d = 1$ case, where g^2 plays the role of $e^3 B$.

The Coulomb potential in $d = 3$ in strong B 3

In the opposite case of superstrong magnetic fields $B \gg 3\pi m_e^2/e^3$ we get:

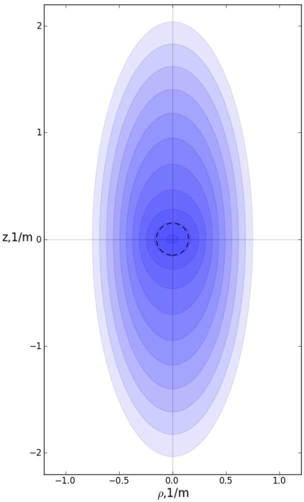
$$\Phi(z) = \begin{cases} \frac{e}{|z|} e^{(-\sqrt{(2/\pi)e^3 B}|z|)}, \frac{1}{\sqrt{(2/\pi)e^3 B}} \ln\left(\sqrt{\frac{e^3 B}{3\pi m_e^2}}\right) > |z| > \frac{1}{\sqrt{eB}} \\ \frac{e}{|z|}(1 - e^{(-\sqrt{6m_e^2}|z|)}), \frac{1}{m_e} > |z| > \frac{1}{\sqrt{(2/\pi)e^3 B}} \ln\left(\sqrt{\frac{e^3 B}{3\pi m_e^2}}\right) \\ \frac{e}{|z|} \quad , \qquad |z| > \frac{1}{m_e} \end{cases} ,$$

$$V(z) = -e\Phi(z)$$



Modified Coulomb potential at $B = 10^{17}$ *G (blue) and its long distance (green) and short distance (red) asympotics.*

picture 2



hydrogen spectrum, odd states

Electrons from LLL are nonrelativistic and move along B; the effective potential is symmetric under $z \to -z$.
The energies of the odd states are:

$$E_{\text{odd}} = -\frac{m_e e^4}{2n^2} + O\left(\frac{m_e^2 e^3}{B}\right) \ , \quad n = 1, 2, \ldots \quad .$$

So, for the superstrong magnetic fields $B \sim m_e^2/e^3$ the deviations of the odd states from the Balmer series are negligible.

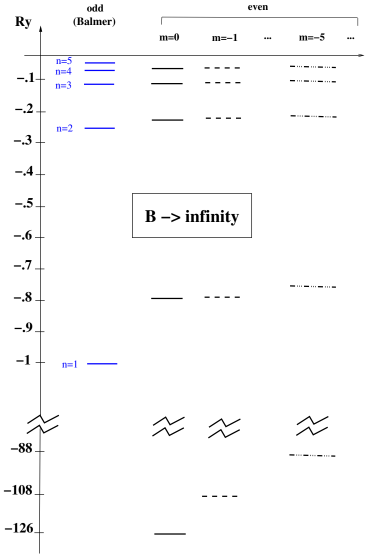
hydrogen spectrum, even states

$$\ln\left(\frac{B}{m_e^2 e^3 + \dfrac{e^6}{3\pi}B}\right) = \lambda + 2\ln\lambda + 2\psi\left(1 - \frac{1}{\lambda}\right) + \ln 2 + 4\gamma + \psi(1+|m|)$$

$E = -(m_e e^4/2)\lambda^2$, for $B \to \infty$: $\lambda \to 11.2$, $E_0 \to -1.7KeV$

Freezing of ground level was discovered by Shabad, Usov (2007, 2008).

hydrogen spectrum



Spectrum of hydrogen levels in the limit of the infinite magnetic field

$Z_{critical}$

When Z grows the ground level goes down and at $Z \approx 170$ it reaches lower continuum, $\varepsilon_0 = -m_e$. Two e^+e^- pairs are produced from vacuum. Electrons with the opposite spins occupy the ground level, while positrons are emitted to infinity.
The magnetic field squeezes the electron wave function in transverse plain making the Coulomb problem one dimensional. So, criticality is reached at smaller Z.
Without screening: V.N. Oraevskii, A.I. Rez, V.B. Semikoz, 1977.
The bispinor which describes an electron on LLL is:

$$\psi_e = \begin{pmatrix} \varphi_e \\ \chi_e \end{pmatrix} \quad ,$$

$$\varphi_e = \begin{pmatrix} 0 \\ g(z) \exp\left(-\rho^2/4a_H^2\right) \end{pmatrix} \quad , \chi_e = \begin{pmatrix} 0 \\ if(z) \exp\left(-\rho^2/4a_H^2\right) \end{pmatrix} \quad .$$

Dirac equation

Dirac equations for functions $f(z)$ and $g(z)$ look like:

$$
\begin{aligned}
g_z - (\varepsilon + m_e - \bar{V})f = 0 \quad , \\
f_z + (\varepsilon - m_e - \bar{V})g = 0 \quad ,
\end{aligned}
$$

where $g_z \equiv dg/dz$, $f_z \equiv df/dz$. They describe the electron motion in the effective potential $\bar{V}(z)$:

$$
\bar{V}(z) = \frac{1}{a_H^2} \int\limits_0^\infty V(\sqrt{\rho^2 + z^2}) \exp\left(-\frac{\rho^2}{2a_H^2} \right) \rho d\rho
$$

$\varepsilon_0^{fr}(Z)$

Z	$\left(E_0^{fr}\right)_{Schr}^{numerical}$, keV	$\left(\varepsilon_0^{fr} - m_e\right)_{Dirac}^{numerical}$, keV
1	-1.7	-1.7
10	-88	-87
20	-288	-273
30	-582	-519
40	-966	-787
49	-	-1003

Table: Values of the freezing ground state energies for different Z from the Schrödinger and the Dirac equations. In order to find the freezing energies we take $B/B_0 = 10^8$.

Z_{cr} without screening

ORS (1977):

$$\frac{B}{B_0} = 2(Z_{cr}e^2)^2 \exp\left(-\gamma + \frac{\pi - 2\arg\Gamma(1 + 2iZ_{cr}e^2)}{Z_{cr}e^2}\right).$$

Z_{cr}

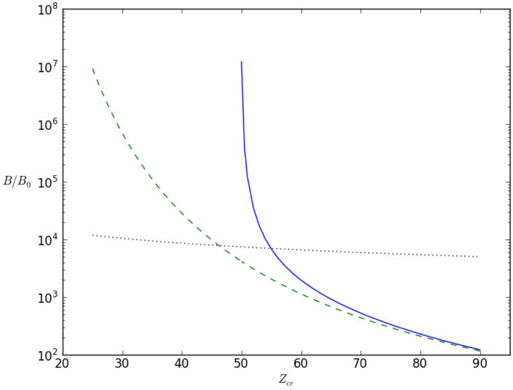


Figure: The values of B_{cr}^{Z}: a) without screening , a dashed (green) line; b) numerical results with screening, a solid (blue) line. The dotted (black) line corresponds to the field at which a_H becomes smaller than the size of the nucleus.

Conclusions

- Spectrum of the levels of a hydrogen atom originating from LLL with the account of screening is found;
- Ground level has finite energy in the limit $B \longrightarrow \infty$;
- The Dirac equation is solved numerically with the account of screening;
- Ions with $Z < 50$ never become critical.