

# Electroweak radiative corrections and heavy top

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# Lecture 1: 5 steps

Why are we going to speak about electroweak radiative corrections (ERC) this summer?

Practical aspects: SM prediction of the value of  $M_H$   
from ERC:  $M_H = (85 + 30 - 20) \text{ GeV}$ .

If LHC discovers a heavy higgs boson, it will mean that new electroweak nonsinglet particle(s) do exist.

Besides higgs: if other particle(s) are discovered at LHC – their contribution(s) to ERC will be one of the first questions you would like to analyze.

So: ERC will be a hot topic at LHC.

Theoretical aspect: creation of the *renormalizable* theory of weak interactions in the 60's is one of the greatest achievements of theoretical physics in the XX century.

So: an educated person should know how to calculate *radiative corrections* in GWS theory.

# why at “Heavy Quark Physics”?

Usually (QED, QCD) heavy particle contributions to rad.corr. are damped.

Muon magnetic moment

$$\mu = e/(2m_\mu)[1 + \alpha/(2\pi)(Schwinger) + \dots + (m_\mu/\Lambda_0)^2(Berestetskii, Krohin, Khlebnikov, 1956)].$$

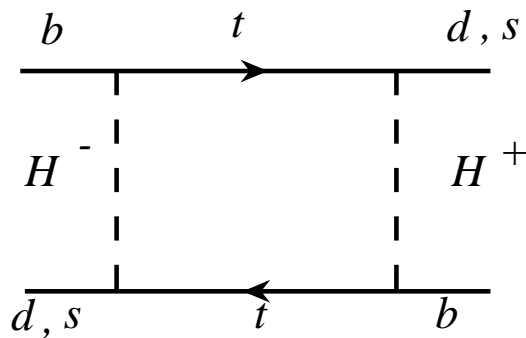
By the way,

$\mu^{exp} = [1 + (1165920.8 \pm 0.6)10^{-9}]e/(2m_\mu)$ , and experimental uncertainty corresponds to  $\Lambda_0 > 3 \text{ TeV}$ ,

LHC?

# nondecoupling

In QAD heavy particle contributions to rad.corr. are enhanced because of Higgs mechanism of mass generation.



t'Hooft-Landau (or any other  $R_\xi$ ) gauge,  $G_H = 1/p^2$

$$(m_t/\eta)^4 \int d^4p p p_\alpha p_\beta / [(p^2 - m_t^2)^2 p^4] =$$
$$\sim m_t^2/\eta^4 \sim G_F^2 m_t^2$$

# what observables?

$$M_Z = 91.188(2) \text{ GeV}; \quad M_W = 80.400(25) \text{ GeV};$$

$$\Gamma_Z, \quad Br(Z \rightarrow l^+l^-), \quad A_{FB}^{l,c,b}$$

and other parameters of  $Z$  are measured now with the precision better than 0.1 % .

How large are ew rad.corr.?

$$\delta \sim \frac{g^2}{16\pi^2} = \frac{\alpha_W}{4\pi} = \frac{\alpha}{4\pi \sin^2 \theta} \approx 0.2\% \implies$$

one needs to use ew radcorrections to deal with these data.

# rad. corr. in QED

QED:  $L(e_0, m_0)$

At one loop we get:

$$e = e_0 \left[ 1 + c_e \frac{\alpha}{\pi} \ln \frac{\Lambda^2}{m_e^2} \right], \quad m = m_0 \left[ 1 + c_m \frac{\alpha}{\pi} \ln \frac{\Lambda^2}{m_e^2} \right], \quad (1)$$

where  $\Lambda$  is the ultraviolet cutoff.

$e$  and  $m$  are measured with record precision and from (1) we get:

$$e_0 \equiv e_0(\Lambda, m, e), \quad m_0 \equiv m_0(\Lambda, m, e).$$

The next step is the calculation of some amplitude; say Compton scattering,  $e\gamma \rightarrow e\gamma$ :

# Compton

$$A = A_0(e_0, m_0) + A_1(e_0, m_0, \Lambda) = A(e, m, s, t)$$

In this way we get a finite expression with one loop rad. corr. taken into account.

QED (quod erat demonstrandum).



$$SU(2)_L \otimes U(1)$$

The situation differs from QED.

Let us consider a gauge sector:

1.  $L(g, g', \eta)$ . These coupling constants and Higgs expectation value are not measured directly and are known with rather poor precision;
2.  $M_Z$  is known precisely, but it is not a parameter of the Lagrangian...

So, some modifications are needed.

# LEPTOP

Different approaches to study rad. corr. are possible.

1989 - start of SLC and LEP I;

1991 - 1995

Victor Novikov, Lev Okun, Alexander Rozanov, M.V.

ZFITTER (D.Yu. Bardin et al., Dubna - Zeuten) - was widely used by LEP collaborations to deal with raw data.

# 5 steps to heaven

1. The best measured observables:  $G_\mu, m_Z, \alpha$

2.  $G_\mu = G_\mu(g_0, \bar{g}_0, \eta_0; \Lambda), \quad m_Z = \dots, \quad \alpha = \dots$

3.  $g_0 = g_0(G_\mu, m_Z, \alpha; \Lambda); \quad \bar{g}_0 = \bar{g}_0(G_\mu, m_Z, \alpha; \Lambda);$   
 $\eta_0 = \eta_0(G_\mu, m_Z, \alpha; \Lambda)$

4.  $m_W = m_W(g_0, \bar{g}_0, \eta_0; \Lambda),$

5.  $m_W =$   
 $m_W[g_0(G_\mu, m_Z, \alpha; \Lambda), \bar{g}_0(G_\mu, m_Z, \alpha; \Lambda), \eta_0(G_\mu, m_Z, \alpha; \Lambda);$   
 $\Lambda]$

Dependence on  $\Lambda$  in the last expression cancels  
because of renormalizability of the theory.

Take other observables

( $\Gamma_Z = \dots$ ,  $A_{FB} = \dots$ , ...)

and repeat items 4 and 5.

This is all what is needed to take into account ew rad. corr. at one loop.

QED.

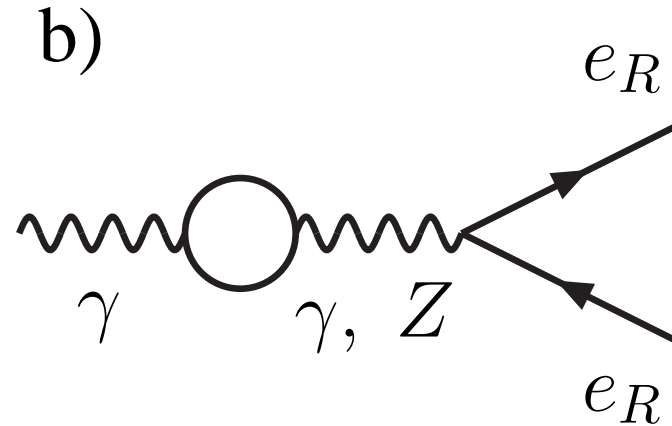
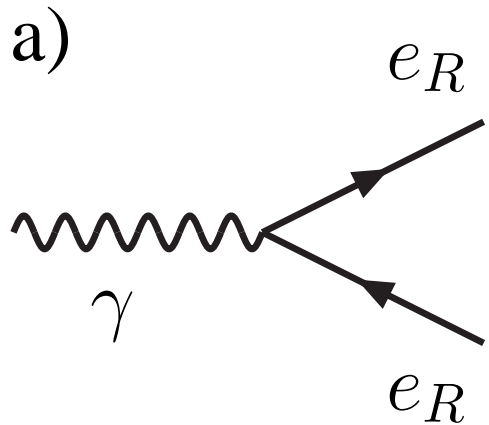
A technical remark: ultraviolet cutoff  $\Lambda$  breaks local gauge invariance. To restore it in QED one subtracts photon mass, which appears to be proportional to  $e\Lambda$ . In QAD we wish to calculate IVB masses.

The way out: dimensional regularization.

We will calculate integrals in  $D = 4 - 2\varepsilon$  dimensional space-time, where they converge and local gauge invariance is not spoiled.

So, in all formulas instead of  $\Lambda$  poles  $1/\varepsilon$  will occur. In final formulas which express physical quantities ( $M_W, \Gamma_Z, \dots$ ) through  $G_\mu, m_Z, \alpha$  these poles cancel.

## step 2, $\alpha$



$$\alpha = \frac{e_0^2}{4\pi} \left[ 1 - \Pi'_\gamma(0) - 2 \frac{s}{c} \frac{\Pi_{\gamma Z}(0)}{M_Z^2} \right] , \quad (2)$$

where  $Z$  interaction equals

$$(T_3 - Qs^2)\bar{g} = -Q\frac{s}{c}e, \quad \bar{g} = g/c = e/(cs)$$

$s \equiv \text{Sin}(\theta)$  - the sine of electroweak mixing angle.

$$\alpha \rightarrow \bar{\alpha}$$

The obtained equation for fine structure constant can be used to get formulas for electroweak rad.corr.

$\Pi'_\gamma(0) \sim \alpha \ln(\Lambda^2/m_{l,q}^2)$ , where  $m_l$  and  $m_q$  are the masses of charged leptons and quarks.

In final expressions u.v. cutoff  $\Lambda$  will be substituted by  $M_Z$ , and logarithmically enhanced rad. corr. will emerge.

Their physical sense is transparent: they correspond to  $\alpha$  running from  $q^2 = 0$  to the electroweak scale  $q^2 = M_Z^2$ .

It is very convenient to take this running into account from the very beginning, separating it from proper weak rad. corr.

$$\bar{\alpha} \equiv \alpha(M_Z) = \frac{e_0^2}{4\pi} \left[ 1 - \frac{\Pi_\gamma(M_Z^2)}{M_Z^2} - 2 \frac{s}{c} \frac{\Pi_{\gamma Z}(0)}{M_Z^2} \right] \quad (3)$$

This equation will be used to determine the bare parameters of electroweak Lagrangian (remember that  $e_0^2 = g_0^2(1 - \frac{g_0^2}{\bar{g}_0^2})$ ).

From Eqs (2,3) one should find the numerical value of  $\bar{\alpha}$ :

$$\bar{\alpha} = \frac{\alpha}{1 - \delta\alpha}, \quad \delta\alpha = \Pi'_\gamma(0) - \frac{\Pi_\gamma(M_Z^2)}{M_Z^2}, \quad (4)$$



and for electron loop one easily obtains:

$$\delta\alpha = \frac{\alpha}{3\pi} \ln\left(\frac{M_Z^2}{m_e^2}\right) .$$

Substituting it into Eq.(4) we obtain one of the most famous equations in physics: zero charge formula of Landau, Abrikosov, Khalatnikov.

Summing up leptonic and hadronic contributions we get:

$$\alpha(M_Z) \equiv \bar{\alpha} = [128.95(5)]^{-1}$$

instead of  $\alpha = [137.0359991(5)]^{-1}$ .

## step 2, $M_Z$



$$G_{\mu\nu}^Z = \frac{-ig_{\mu\nu}}{k^2 - M_{Z_0}^2 + \Pi_Z(k^2)} + \dots ,$$

The pole position corresponds to the  $Z$ -boson mass:

$$M_Z^2 = M_{Z_0}^2 - \Pi_Z(M_Z^2) , \quad M_{Z_0} = \frac{\bar{g}_0 \eta_0}{2} . \quad (5)$$

## step 2, $G_\mu$

$$\mu \rightarrow e\nu_\mu\bar{\nu}_e$$

It is convenient to divide rad. corr. into 2 parts: dressing of  $W$  - boson propagator, described by  $\Pi_W(0)$ , and vertexes and boxes, denoted by  $D$ :

$$\frac{G_\mu}{\sqrt{2}} = \frac{g_0^2}{8m_{W_0}^2} \left[ 1 + \frac{\Pi_W(0)}{M_W^2} + D \right] = \frac{1}{2\eta_0^2} \left[ 1 + \frac{\Pi_W(0)}{M_W^2} + D \right] . \quad (6)$$

What about logarithmic running of the weak charge from  $q^2 \approx m_\mu^2$  to  $q^2 \approx M_W^2$ ?

$\Pi_W(q^2)$  contains logarithmic term:

$\Pi_W(q^2) \sim q^2 \ln \frac{\Lambda^2}{\max(q^2, m_e^2)}$ . However, due to nonzero mass of IVB running takes place only above this mass.

So, there are 2 conditions for the charge to run logarithmically:

momentum transfer should be larger than the masses of the particles in the loop *and* larger than the mass of the corresponding vector boson.

Or the distances should be smaller...

That is why in the  $Z$  - and  $W$  - boson physics the big log occurs only in the running of  $\alpha$ .

## step 3

$$G_\mu = \frac{1}{\sqrt{2}\eta_0^2} \left[ 1 + \frac{\Pi_W(0)}{M_W^2} + D \right] ,$$

$$M_Z^2 = \frac{1}{4} \bar{g}_0^2 \eta_0^2 - \Pi_Z(M_Z^2) ,$$

$$4\pi\bar{\alpha} = g_0^2 \left( 1 - \frac{g_0^2}{\bar{g}_0^2} \right) \left[ 1 - \frac{\Pi_\gamma(M_Z^2)}{M_Z^2} - 2 \frac{s}{c} \frac{\Pi_{\gamma Z}(0)}{M_Z^2} \right]$$

.

For bare parameters we get:

$$\eta_0^2 = \frac{1}{\sqrt{2}G_\mu} \left[ 1 + \frac{\Pi_W(0)}{M_W^2} + D \right] ,$$

$$\bar{g}_0^2 = 4\sqrt{2}G_\mu M_Z^2 \left[ 1 + \frac{\Pi_Z(M_Z^2)}{M_Z^2} - \frac{\Pi_W(0)}{M_W^2} - D \right] ,$$

and it is convenient to rewrite the equation for  $g_0$  in the following way:

$$\frac{g_0^2}{\bar{g}_0^2} \left( 1 - \frac{g_0^2}{\bar{g}_0^2} \right) = \frac{\pi \bar{\alpha}}{\sqrt{2}G_\mu M_Z^2} \left( 1 + \frac{\Pi_W(0)}{M_W^2} - \frac{\Pi_Z(M_Z^2)}{M_Z^2} + \frac{\Pi_\gamma(M_Z^2)}{M_Z^2} + \right. \\ \left. + 2 \frac{s}{c} \frac{\Pi_{\gamma Z}(0)}{M_Z^2} + D \right) .$$

$\sin\theta$

Arithmetic was enough to solve the first 2 equations;  
for the third one trigonometry is needed.

Let us define an electroweak mixing angle:

$$\sin^2 \theta \cos^2 \theta = \frac{\pi \bar{\alpha}}{\sqrt{2} G_\mu M_Z^2}, \quad \sin^2 \theta = 0.2310(1)$$

and solve the third equation:

$$\frac{g_0}{\bar{g}_0} = c \left[ 1 + \frac{s^2}{2(c^2 - s^2)} \left( \frac{\Pi_Z(M_Z^2)}{M_Z^2} - \frac{\Pi_W(0)}{M_W^2} - \frac{\Pi_\gamma(M_Z^2)}{M_Z^2} - 2 \frac{s}{c} \frac{\Pi_{\gamma Z}(0)}{M_Z^2} - D \right) \right].$$

## step 4; custodial symmetry

Let us start from the  $W$  - boson mass:

$$M_W^2 = M_{W_0}^2 - \Pi_W(M_W^2) , \quad M_{W_0} = \frac{g_0 \eta_0}{2} .$$

At step 2 an analogous equation was written for  $M_Z$ ; using it we get:

$$\frac{M_W}{M_Z} = \frac{g_0}{\bar{g}_0} \left[ 1 + \frac{\Pi_Z(M_Z^2)}{2M_Z^2} - \frac{\Pi_W(M_W^2)}{2M_W^2} \right] .$$

If  $U(1)$  charge  $g'_0$  were zero, then  $g_0 = \bar{g}_0$ , and at tree level  $M_W = M_Z$ , which is a good approximation to the real life:  $80 GeV \approx 90 GeV$ .



## *t*: anticutodial symmetry

What about loops? If  $m_{up} = m_{down}$ , then

$\Pi_Z(M_Z^2) = \Pi_W(M_W^2)$  and IVB stay degenerate.

In the real life top quark is extremely heavy, and the contribution of the  $(t, b)$  doublet to the difference of  $W$ - and  $Z$ - boson masses is enhanced as  $m_t^2/M_Z^2 \approx 4$ .

## step 5, $M_W$

$$\begin{aligned} \frac{M_W}{M_Z} = & c + \frac{c^3}{2(c^2 - s^2)} \left( \frac{\Pi_Z(M_Z^2)}{M_Z^2} - \frac{\Pi_W(M_W^2)}{M_W^2} \right) + \\ & + \frac{cs^2}{2(c^2 - s^2)} \left( \frac{\Pi_W(M_W^2)}{M_W^2} - \frac{\Pi_W(0)}{M_W^2} - \frac{\Pi_\gamma(M_Z^2)}{M_Z^2} - \right. \\ & \left. - 2 \frac{s}{c} \frac{\Pi_{\gamma Z}(0)}{M_Z^2} - D \right) , \end{aligned}$$

UV divergences cancel in the last expression:  
*the formula for finite one loop ew rad. corr. to the  
ratio of IVB masses is obtained!!!*

$$Z \rightarrow l^+ l^-$$

Reminding that  $Z$ -boson coupling constant is  $\bar{g}_0$  and corresponding generator equals

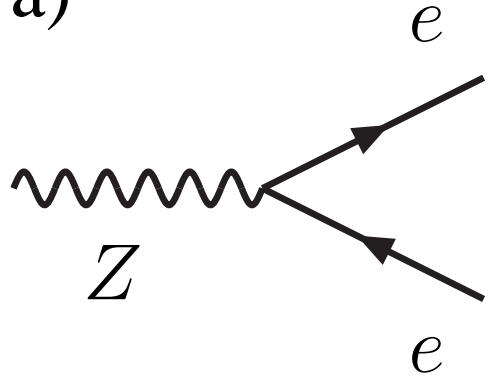
$$T_3 - s_0^2 Q$$

we get for the decay amplitude at the tree level:

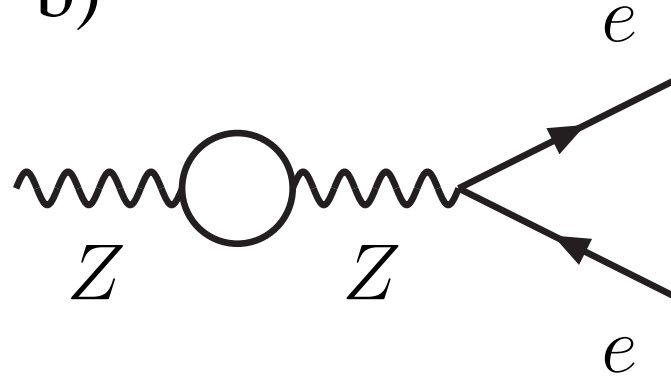
$$A_0 = \frac{\bar{g}_0}{2} \bar{l} \left[ -\frac{1}{2} \gamma_\alpha \gamma_5 - \left( \frac{1}{2} - 2s_0^2 \right) \gamma_\alpha \right] l Z_\alpha .$$

Taking into account the expressions for  $\bar{g}_0$  and  $g_0/\bar{g}_0$  as well as the loop corrections to the tree diagram (see next slide) we straightforwardly obtain the expression for the decay amplitude free from the ultraviolet divergences.

a)



b)



c), d),...

$A_{ll}$

$$\begin{aligned}
 A = & \sqrt{\sqrt{2}G_\mu M_Z^2} \left[ 1 + \frac{\Pi_Z(M_Z^2)}{2M_Z^2} - \frac{\Pi_W(0)}{2M_W^2} - \frac{1}{2}D - \right. \\
 & \left. - \frac{1}{2}\Pi'_Z(M_Z^2) \right] \times \left[ \left(-\frac{1}{2} + F_A\right) \bar{l} \gamma_\alpha \gamma_5 l + \right. \\
 & \left. + \left( 2s^2 - \frac{1}{2} + F_V + 2cs \frac{\Pi_{Z\gamma}(M_Z^2)}{M_Z^2} + \frac{2s^2 c^2}{c^2 - s^2} \times \right. \right. \\
 & \left. \left. \times \left( \frac{\Pi_W(0)}{M_W^2} - \frac{\Pi_Z(M_Z^2)}{M_Z^2} + \frac{\Pi_\gamma(M_Z^2)}{M_Z^2} + 2 \frac{s}{c} \frac{\Pi_{\gamma Z}(0)}{M_Z^2} + D \right) \right) \right. \\
 & \left. \bar{l} \gamma_\alpha l \right]
 \end{aligned}$$

## step 5, $g_A$ and $g_V$

Let us rewrite the expression for the decay amplitude:

$$A = \sqrt{\sqrt{2}G_\mu M_Z^2 \bar{l} [g_A \gamma_\alpha \gamma_5 + g_V \gamma_\alpha] l Z_\alpha} .$$

The UV finite expressions for axial and vector coupling constants are given by a long formula on a previous slide.

# Lecture 2: top and higgs

Formulas for electroweak radiative corrections obtained in the first lecture are characterized by strong dependence on the top quark mass which helped to find the top at TEVATRON in 1994.

There are 2 places in ew corrections to IVB parameters where top quark contributions are enhanced:

1. the polarization operators (of nonconserved currents);
2.  $Z \rightarrow t\bar{t} \rightarrow b\bar{b}$  decay amplitude.

Why does current nonconservation matter?

$\Pi_\gamma(q^2), \Pi_{\gamma Z}(q^2) \sim [g_{\mu\nu}q^2 - q_\mu q_\nu](a + bq^2/m_t^2 + \dots)$ ,  
while  $\Pi_W \sim g_{\mu\nu}(m_t^2 + \dots)$ .

## $m_t^2$ term from $Pi$ 's

In the limit  $m_t^2 \gg m_W^2, m_Z^2$  we have:

$\Pi(m_V^2) = \Pi(0)$ , that is why we get the following relations:

$$\frac{M_W}{M_Z} = c + \frac{c^3}{2(c^2 - s^2)} \left( \frac{\Pi_Z(0)}{M_Z^2} - \frac{\Pi_W(0)}{M_W^2} \right) ,$$

$$g_A = -\frac{1}{2} - \frac{1}{4} \left( \frac{\Pi_Z(0)}{M_Z^2} - \frac{\Pi_W(0)}{M_W^2} \right) ,$$

$$g_V/g_A = 1 - 4s^2 + \frac{4c^2 s^2}{c^2 - s^2} \left( \frac{\Pi_Z(0)}{M_Z^2} - \frac{\Pi_W(0)}{M_W^2} \right) .$$

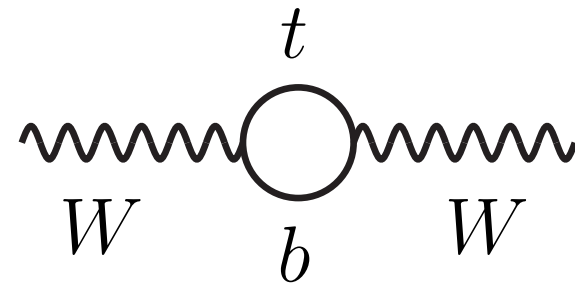
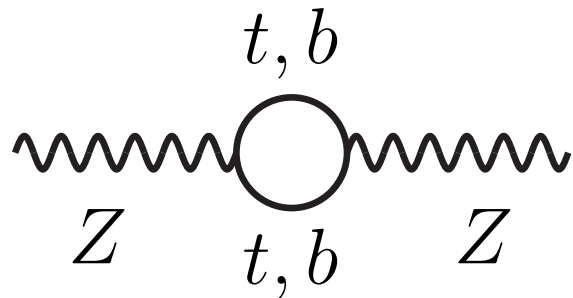


In order to honestly calculate the  $m_t$  dependence of the physical observables one should calculate the top quark contributions to polarization operators:

$$-i\Pi_{\mu\nu}^{\psi}(q^2) = -$$

$$- \int \frac{d^D k}{(2\pi)^D \mu^{D-4}} \frac{Sp \gamma_{\mu}(\gamma_5)(\hat{k} + m_1)\gamma_{\nu}(\gamma_5)(\hat{k} + \hat{q} + m_2)}{(k^2 - m_1^2)((k + q)^2 - m_2^2)},$$

where we use dimensional regularization of the quadratically divergent expression:  $D = 4 - 2\varepsilon$  and the factor  $\mu$  takes care of the canonical dimension of the integral.

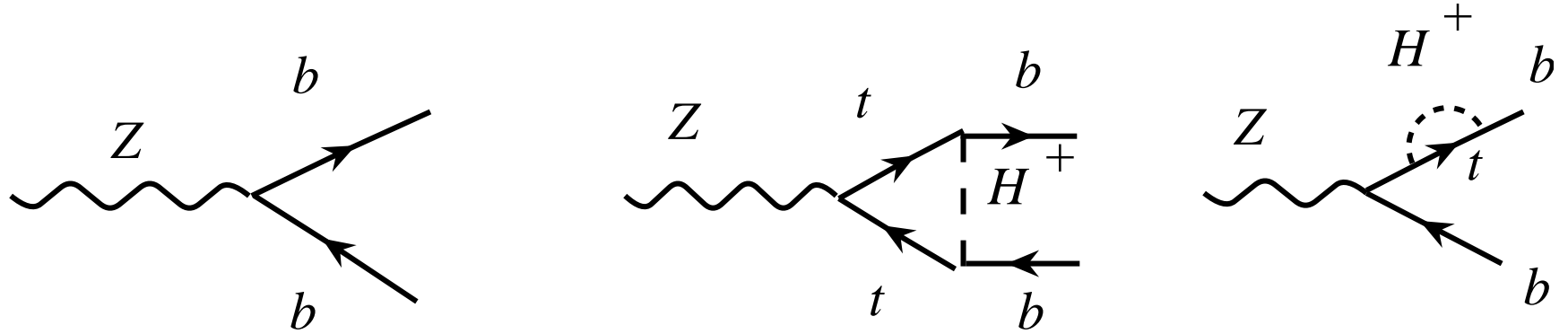


“back of the envelope” calculation of  $m_t^2$  term:

$$\frac{\Pi_Z^\psi(0)}{M_Z^2} - \frac{\Pi_W^\psi(0)}{M_W^2} = \frac{3\bar{\alpha}}{8\pi c^2 s^2 M_Z^2} \int \frac{dp^2}{(p^2 + m_t^2)^2} \times$$

$$\begin{aligned} & \times \left[ p^4/2 + (p^2 + m_t^2)^2/2 - p^2(p^2 + m_t^2) \right] = \\ & = \frac{3\bar{\alpha}}{16\pi c^2 s^2} \left( \frac{m_t}{M_Z} \right)^2 . \end{aligned}$$

# specific for $Z \rightarrow bb$ decay $m_t^2$ term



Htb vertex is proportional to  $m_t$ , that is why one-loop diagrams produce correction to  $Zbb$  coupling enhanced as  $(m_t/M_Z)^2$ .

To calculate this correction we can neglect  $Z$ -boson momentum.

$Z$ -boson coupling is proportional to  $T_3 - Qs^2$ . The part proportional to  $b$  - quark electric charge  $Q$  induces vector coupling which is not renormalized by higgs loop (CVC) - so, at zero momentum transfer ( $q_\mu^Z = 0$ ) the sum of one loop diagrams is zero.

What remains is  $T_3$  which induces the coupling with  $b_L$  and  $t_L$ .

And again the vector part is not renormalized, so only axial current remains.

Since  $Z$ -boson has only vector coupling with higgs, we should not calculate a corresponding vertex diagram. And there is only the diagram with  $Ztt$  coupling to be taken into account.

Calculating a vertex diagram with  $\Lambda$  UV cutoff we get:

$$\bar{g}/4/(16\pi^2)\left(\frac{m_t}{\eta/\sqrt{2}}\right)^2 \times$$

$$\times \left[-1/2\ln(\Lambda^2/m_t^2) + 3/2\right] \bar{b}\gamma_\alpha \frac{(1 + \gamma_5)}{2} b Z_\alpha ,$$

while  $(tH)$  insertions into external legs give:

$$\bar{g}/4/(16\pi^2)\left(\frac{m_t}{\eta/\sqrt{2}}\right)^2 \times$$

$$\times \left[1/2\ln(\Lambda^2/m_t^2) + 1/2\right] \bar{b}\gamma_\alpha \frac{(1 + \gamma_5)}{2} b Z_\alpha .$$

The sum of the two last expressions produces correction to  $g_L^b$ :

$$\begin{aligned}
 & -\bar{g}/2\left[1 - \left(\frac{m_t}{\eta/\sqrt{2}}\right)^2/(16\pi^2)\right]\bar{b}\gamma_\alpha\frac{(1+\gamma_5)}{2}b Z_\alpha = \\
 & = -\bar{g}/2\left[1 - \frac{\alpha}{8\pi c^2 s^2}\left(\frac{m_t}{M_Z}\right)^2\right]b\gamma_\alpha\frac{(1+\gamma_5)}{2}b Z_\alpha ,
 \end{aligned}$$

which reduces  $\Gamma_Z(bb)$ .

Experimental measurement...

# $M_H$

Electroweak rad. corr. depend on  $M_H$ . This is the reason why from precision measurement of  $Z$ - and  $W$ -boson parameters the value of  $M_H$  is extracted.

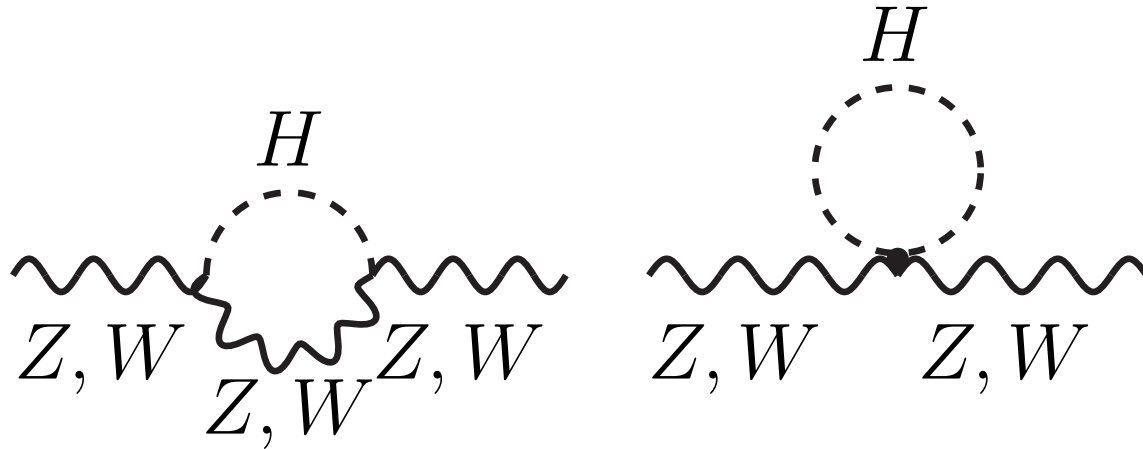
Which diagrams matter? The radiation of higgs from the fermion line is proportional to  $m_f/\eta$ , and since

$\eta = 1/(\sqrt{\sqrt{2}G_\mu}) = 246\text{GeV}$  even in the case of  $b$ -quark it should be neglected.

What remain are the vector boson polarization operators (just as in the case of top, if we forget for the moment  $Z \rightarrow bb$  decay).

It is convenient to perform calculations in unitary gauge where the nonphysical degrees of freedom ( $H^\pm, ImH^0$ ) are absent.





$$\begin{aligned}
 & \left(\frac{g\eta}{2}\right)^2 |W|^2 \rightarrow \left(\frac{g(\eta + H^0)}{2}\right)^2 |W|^2 = \\
 & = \left(\frac{g\eta}{2}\right)^2 |W|^2 + \frac{1}{2}g^2\eta H^0 |W|^2 + \frac{g^2}{4}H^{0^2} |W|^2 = \\
 & = M_W^2 |W|^2 + gM_W H^0 |W|^2 + \frac{1}{4}g^2 H^{0^2} |W|^2
 \end{aligned}$$

Analogously from the  $Z$ -boson mass term in Lagrangian we can obtain  $HZZ$  and  $HHZZ$  coupling constants.

$$H \rightarrow \gamma\gamma$$

A less trivial example:

$$L_{eff}(q^2 \ll M^2) = \frac{1}{4} F_{\mu\nu}^2 \times \\ \times \sum \frac{b_i e^2}{16\pi^2} \log\left(\frac{\Lambda^2}{M_i^2}\right) ,$$

where  $b = -4/3$  for a charged lepton,  $-4Q^2$  for a quark, 7 for  $W$ -boson are the coefficients of Gell-Mann – Low function. Substituting  $M = f\eta(1 + H/\eta)$  and expanding the logarithm we obtain the amplitude of  $H \rightarrow \gamma\gamma$  decay. The most remarkable in the last formula is the sign of the  $W$  loop contribution, opposite to that of the lepton and quark loop, and number 7 as well.

# asymptotic freedom in the USSR, 1965

Let me start from number 7 obtained by V.S.Vanyashin and M.V.Terentyev in their 1965 ZhETPh paper. At present the easiest way to derive it is the following:

$7 = 11/3C_V - 1/6 - 1/6$ ,  $C_V = 2$  for  $SU(2)$  ,  
where  $11/3C_V$  is the contribution of the massless vectors in adjoint representation to the  $\beta$ -function. One factor  $1/6$  comes from the higgs doublet contribution to the same  $SU(2)$   $\beta$ -function, while another  $1/6$  is the higgs doublet contribution to the running of the coupling constant  $g'$ ,

$$1/e = 1/g + 1/g' .$$

Concerning the sign Vanyashin and Terentiev stressed that it is opposite to that which *always* occurs in QFT.

A bit of history: I heard from Terentiev that when he was making a report at ITEP seminar on this paper Pomeranchuk said that evidently the theory was not selfconsistent (he relied upon Landau - Pomeranchuk “zero-charge”). At the end of the paper this “wrong sign” behavior of  $\beta$ -function was attributed to nonrenormalizability of the electrodynamic of massive charged vector bosons.

However in the Abstract the anomalous character of the electric charge renormalization was emphasized.

M.V. Terentiev worked at ITEP, V.S. Vanyashin worked and is still working at Dnepropetrovsk Physico-Technical Institute.

It is remarkable that if higgs boson mass is around 120 GeV (which is quite probable: SM fit + recent Tevatron results), than  $H \rightarrow \gamma\gamma$  decay will play the important role in higgs discovery and factor “7” will become known to everybody in hep community 45 years after its first appearance.

# back to higgs in rad.corr.

Calculation of  $W$ - and  $Z$ -boson polarization operators provides us with explicit dependence of physical observables on  $M_H$ . In the limit  $M_H \gg M_Z$  we get:

$$\Pi_W^H \sim M_H^2 \ln(M_H^2) + M_H^2 + (M_W^2 + q^2) \ln(M_H^2) .$$

In the differences of polarization operators on which physical quantities depend:

$$\frac{\Pi_W(M_W^2)}{M_W^2} - \frac{\Pi_W(0)}{M_W^2} , \quad \frac{\Pi_Z(M_Z^2)}{M_Z^2} - \frac{\Pi_W(M_W^2)}{M_W^2} \quad \text{and}$$

$\Pi'_Z(M_Z^2)$  the first two terms cancel and we are left with the logarithmic dependence on higgs mass .

# asymptotics

$$\frac{M_W}{M_Z} = c + \frac{3\bar{\alpha}c}{32\pi s^2(c^2 - s^2)} \left[ \left(\frac{m_t}{M_Z}\right)^2 - \frac{11}{9}s^2 \ln\left(\frac{M_H}{M_Z}\right)^2 \right],$$

$$g_A = -\frac{1}{2} - \frac{3\bar{\alpha}}{64\pi c^2 s^2} \left[ \left(\frac{m_t}{M_Z}\right)^2 - s^2 \ln\left(\frac{M_H}{M_Z}\right)^2 \right],$$

$$\frac{g_V}{g_A} = 1 - 4s^2 + \frac{3\bar{\alpha}}{4\pi(c^2 - s^2)} \left[ \left(\frac{m_t}{M_Z}\right)^2 - \left(s^2 + \frac{1}{9}\right) \times \right. \\ \left. \times \ln\left(\frac{M_H}{M_Z}\right)^2 \right].$$

Since the coefficients multiplied by log are almost equal, without the knowledge of  $m_t$  one could not determine  $M_H$ .



# Lecture 3: SM fits; NP contributions

After top discovery at Tevatron in 1994 the electroweak precision data provide information on higgs mass.

The dependence on  $M_H$  is provided by  $P_i$ 's; the “constants” are also very important. The expressions in square brackets at a previous slide are substituted by three functions:

$$V_m(t, h) , V_A(t, h) , V_R(t, h) ;$$

$$t \equiv (m_t/M_Z)^2, h \equiv (M_H/M_Z)^2,$$

which take into account all the existing loop calculations ( $\alpha_W, \alpha_s \alpha_W, \dots$ , for details see Novikov, Okun, Rozanov, Vysotsky, “LEPTOP”, hep-ph/9503308; Rep.Prog.Phys.62, 1275(1999).

# Yellow Report

After the first years of LEP1 operation it has become clear that the experimental data on  $Z$  parameters will have very high accuracy. That is why 4 codes which existed in literature have been compared with the aims to check numerical consistency of the different approaches to rad. corr. calculation and to determine theoretical uncertainties.

The results are published in the CERN Yellow Report 95-03, Editors D.Bardin, W. Hollik, G.Passarino.

From history to our days.

LEPTOP fit of the precision observables is shown on the next slide (A.Rozanov, summer 2008).

# SM fit by LEPTOP

Observable	Exper. data	LEPTOP fit	Pull
$\Gamma_Z, \text{ GeV}$	2.4952(23)	2.4963(15)	-0.5
$\sigma_h, \text{ nb}$	41.540(37)	41.476(14)	1.8
$R_l$	20.771(25)	20.743(18)	1.1
$A_{\text{FB}}^l$	0.0171(10)	0.0164(2)	0.8
$A_\tau$	0.1439(43)	0.1480(11)	-0.9
$R_b$	0.2163(7)	0.2158(1)	0.7
$R_c$	0.172(3)	0.1722(1)	-0.0
$A_{\text{FB}}^b$	0.0992(16)	0.1037(7)	-2.8
$A_{\text{FB}}^c$	0.0707(35)	0.0741(6)	-1.0
$s_l^2 (Q_{\text{FB}})$	0.2324(12)	0.2314(1)	0.8

Observable	Exper. data	LEPTOP fit	Pull
$A_{LR}$	0.1513(21)	0.1479(11)	1.6
$A_b$	0.923(20)	0.9349(1)	-0.6
$A_c$	0.670(27)	0.6682(5)	0.1
$m_W, \text{ GeV}$	80.398(25)	80.377(17)	0.9
$m_t, \text{ GeV}$	172.6(1.4)	172.7(1.4)	-0.1
$M_H, \text{ GeV}$		$84^{+32}_{-24}$	
$\hat{\alpha}_s$		0.1184(27)	
$1/\bar{\alpha}$	128.954(48)	128.940(46)	0.3
$\chi^2/n_{\text{d.o.f}}$		18.1/12	

With 10 MeV accuracy of  $M_W$  the accuracy in  $M_H$  will be (+20 -15) GeV,

while at the moment  $M_H < 140(150)\text{GeV}$  at 95% C.L.,  $M_H < 185(200)\text{GeV}$  at 99.5% C.L. (THEORETICAL UNCERTAINTY IS TAKEN INTO ACCOUNT).

This is the end of the Standard Model story.

The main results for other domains of particle physics:

1. QCD: power corrections for  $Z$  width into hadrons are definitely negligible; the obtained value of  $\hat{\alpha}_s$  appears to be considerably larger than (some) QCD people believed; in particular  $J/\psi$  is outside of the perturbative QCD domain;
2. GUT: the precise determination of  $\sin\theta$  excludes unification without low energy SUSY.

# New Physics

What if LHC after a couple of months of operation will announce the discovery of 300 GeV (or even heavier) higgs?

It will definitely mean that beyond Standard Model there are other electroweak nonsinglet particles which contribute to the functions  $V_i$  and shift the value of  $M_H$  in the minimum of  $\chi^2$ .

Before discussing New Physics contribution to rad.corr. let me present two popular sets of parameters widely used in literature.

## $\varepsilon_1, \varepsilon_2, \varepsilon_3$

A set of three parameters  $\varepsilon_i$  has been suggested by Altarelli, Barbieri and Jadach for the most general phenomenological analysis of New Physics. These parameters are in one-to-one correspondence with our parameters  $V_i$ :

$$\varepsilon_1 \sim \alpha_W V_A$$

$$\varepsilon_2 \sim \alpha_W [(V_A - V_m) - 2s^2(V_A - V_R)]$$

$$\varepsilon_3 \sim \alpha_W (V_A - V_R)$$

Since  $\varepsilon_2$  and  $\varepsilon_3$  do not contain the leading  $\sim m_t^2$  term, their values were useful in search for New Physics before the mass of top quark was measured directly.

# $S, T, U$

These letters, popular in particle physics, were used by Peskin and Takeuchi for the parametrization of the so-called oblique corrections due to New Physics contribution to electroweak observables.

Schematically:

$$\delta\varepsilon_1 = \alpha T, \quad \delta\varepsilon_2 \sim \alpha U, \quad \delta\varepsilon_3 = \alpha S,$$

where  $\delta$  means that only NP contributions should be taken into account.

Literally, Peskin and Takeuchi made one more step. Discussing NP with a scale much larger than  $M_Z$  they expanded the polarization operators at  $q^2 = 0$ , taking into account only the first two terms,  $\Pi(0)$  and  $\Pi'(0)$ , which is the correct approximation as far as higher derivatives



are suppressed as  $[(M_Z^2)/(M_{NP}^2)]^n$ . One can find in the literature (PDG) the allowed domains of  $S$ ,  $U$  and  $T$  for a given value of higgs mass and check, if your favourite NP model falls in these domains.

However, some caution is necessary:

1. if the mass of a new particle is only slightly above  $M_Z/2$  then the heavy mass expansion does not work;
2. the allowed domain of  $S$ ,  $U$  and  $T$  depends on  $M_H$ .

# to decouple or to nondecouple?

This is the first question you must ask analyzing NP. The most famous example of NP with decoupling is SUSY.

Why do sleptons and squarks decouple? Because mass splitting within  $SU(2)$  doublet is small, while from scalar fields you can organize only vector current, which is conserved. What about charginos and higgsinos? They form vector multiplets (not chiral) which also decouples.

As a result the direct searches of the superpartners push lower limits on their masses so high (hundreds of GeV) that their contributions to rad.corr. are **MOSTLY** negligible.

Is there any relation between low energy SUSY and rad. corr. except SUSY GUT?

Yes: in all the variants of SUSY the lightest higgs boson mass appeared to be less than 200 GeV, usually close to 100 GeV, which nicely coincides with the values of  $M_H$  obtained from rad. corr.

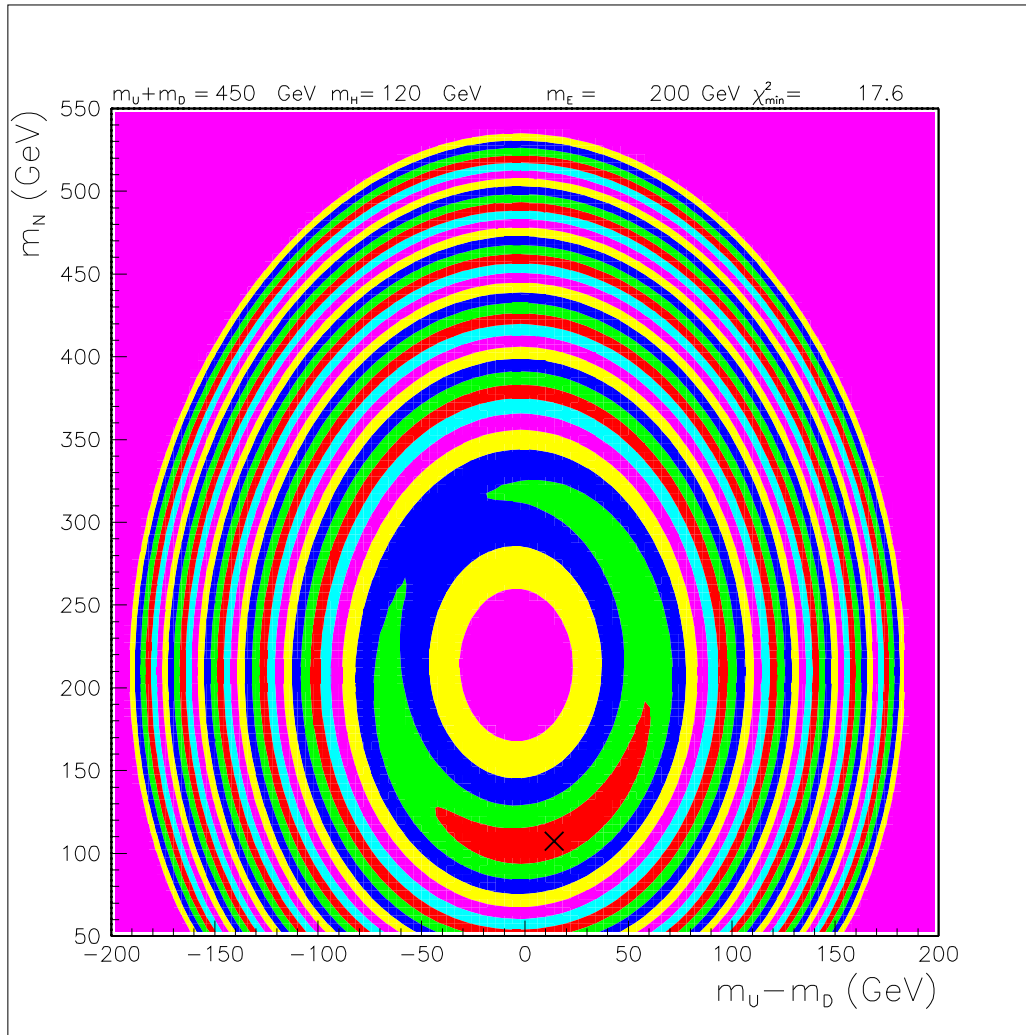
# 4 generation

The simplest example of nondecoupled New Physics. It nondecouples just as the third generation with heavy top. Mass of the neutral lepton  $N$  should be larger than  $M_Z/2$  since  $Z$  boson width allows only 3 light neutrino flavors.

Many new parameters: masses of new particles and their mixing with three light generations. For simplicity let us suppose that mixing is small.

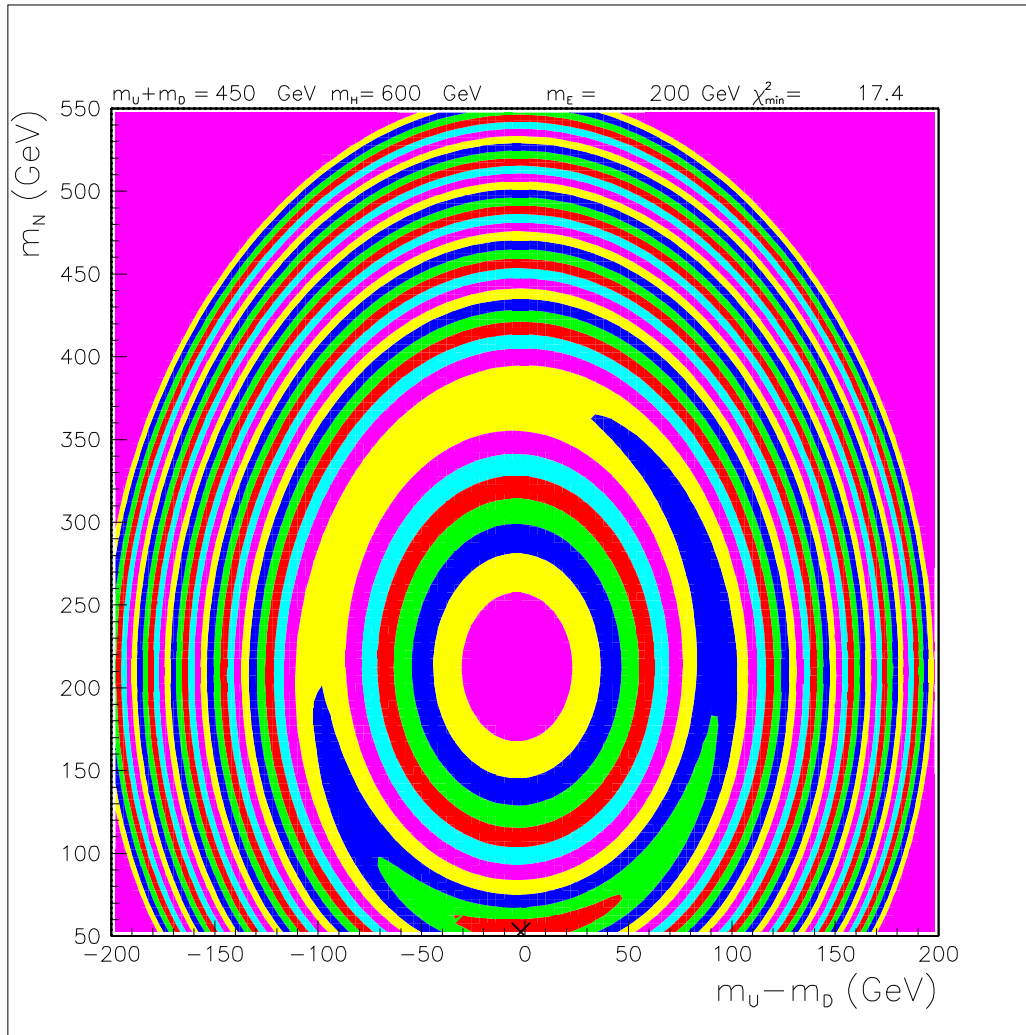
At the next two slides the results of data fit by the LEPTOP code performed by Alexander Rozanov in summer 2008 are presented.

# 4 generation with 120 GeV higgs



$m_E = 200 \text{ GeV}$ ,  
 $m_U + m_D = 450 \text{ GeV}$ ,  $\chi^2/d.o.f. = 17.6/11$ , the  
quality of fit is the same as in SM.

# 4 generation with 600 GeV higgs



$m_E = 200 \text{ GeV}$ ,  
 $m_U + m_D = 450 \text{ GeV}$ ,  $\chi^2/d.o.f. = 17.4/11$ , the  
quality of the fit is the same as in SM.

# Conclusions

Higgs is light **ONLY** in SM or if NP decouples.

Now, in August 2008, protons are approaching LHC detectors.

Probably in half a year or in two years the last pages of Electroweak Interactions will be written.